

Mathematical Modeling of Blood Flow Through an Eccentric Catheterized Artery: A practical approach for a complex system

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Abstract— In this research a two dimensional, single phase, and isothermal model is developed to investigate the effects of eccentric catheterization on blood flow characteristics in a tapered and stenosis artery which is complex system. The model conducted by assuming that the blood is as Newtonian and incompressible fluid and the temperature effects are also neglected. The results clearly show that the axial velocity and the magnitude of the wall shear stress distribution are higher for eccentric catheter than that for concentric one. Also, the resistance impedance gives the reverse trend of the wall shear stress with respect to the taper angle where blood can flow freely through diverging vessel but in the case of eccentric catheter is less than that of the concentric one when the radius of catheter is considered. In addition, the trapping appears near the wall of catheter and the trapped bolus increases in size as the radius of catheter increases.

Keywords- *Mathematical Modeling; Complex systems; Mathematical Biology; Catheter; Blood flow; Eccentric*

I. INTRODUCTION

A catheter is a thin, flexible, hollow and lengthy plastic tube that is generally inserted into an artery (large blood vessel) placed in the wrist; but can also be inserted into the elbow, groin, foot or the inside of the arm. However, in theory, every artery could be used for injection of catheter but in practical situation, arteries of the mentioned organs are commonly placed for the injection. Catheters have an important role in medicine for heart problems. For instance, they are usually used to diagnose and treat heart conditions. During catheterization, small tubes (catheters) are inserted into the circulatory system under x-ray guidance in order to obtain information about blood flow and pressures within the heart and to determine if there are obstructions within the blood vessels feeding the heart muscle (coronary arteries). In addition, a catheter is used for the measurement of various physiological flow characteristics such as arterial blood pressure or pressure gradient and flow velocity or flow rate. Examples include patients in the intensive care unit (ICU) requiring isotropic support or patients with severe cardiovascular disease undergoing surgery. Furthermore, catheter can be used for periodic arterial blood gas analysis in patients with respiratory failure, or severe acid/base

disturbance. When a patient has a lung problem that is so severe it requires checking the levels of oxygen or carbon dioxide of the blood more than 3 to 4 times a day regularly, the arterial catheter is used to draw blood without having to repeatedly stick a needle into the patient body.

When a catheter is inserted into a blood vessel, blood clots can be formed on the tips of arterial catheters, the clots can block blood flow. Furthermore, bleeding can occur at the time of inserting the catheter. Patient becomes uncomfortable resulting from the injection. However, more important, the injection of a catheter alters the flow field and disturbs the hemodynamic conditions in the artery. In practical situation, the injection of a catheter cannot be exactly concentric with the artery. It is in fact placed in an eccentric position. Hence, this study investigates what happen to the blood flow characteristics when catheter is inserted eccentrically through an artery. The main objective of this research is to determine the effects of eccentric catheterization on blood flow characteristics in a tapered and stenosis artery which is belongs to human complex systems; its predicting and calculating the velocity of the blood are very hard due to the fact that they are several parameters take part in this calculations. Specific objectives are to calculate the axial velocity, stream function and trapping, resistance impedance and wall shear stress.

The paper can be outlined as follows: Section II describes the related works chronologically; Section III explains the approach and explained proposed model related to it; we show the results in three important qualities of services in Section IV; finally, in Section V we conclude our work.

II. RELATED WORKS

These papers represent a flavor of the significant ongoing work. There are several researches in this area such as Kanai et al. [1] observed increase of the pressure produced by the insertion of a catheter. In this experimental investigation the results with the data which were obtained from experiments on live dogs have been compared. Blood modeled as Newtonian fluid through co-axial tubes which supposed the outer tube is artery and the inner tube is a catheter. In another study the straight model of artery changed to a curved pipe and studied it with a catheter. Blood supposed as Newtonian fluid. Increases of the shear stress on the artery wall were

calculated. Axial velocity and the streamlines were compared [2]. Dash et al. [3] analyzed a mathematical model of blood flow in a curved artery with catheter and stenosis. They added analytic estimation of a double series perturbation analysis for the case of small curvature and moderate stenosis; and the influence of catheterization on flow characteristics was investigated. Dash et al. [4] studied increasing of flow resistance in catheterized artery by changing Newtonian model of blood flow to modeling it as a Casson fluid. Both of the cases of steady and pulsatile flow situations were studied. The pulsatile flow was analyzed by considering the pressure gradient as a periodic function of time with small inertial effects. The effect of plasticity and characterizes of blood flow was investigated in this research. Modeling of blood flow in another investigation was changed to Herschel-Bulkley fluid and analyzed the influence of the catheter radius and flow rate, wall shear stress and frictional resistance [5]. Two other researchers with assuming blood flow as a two-Newtonian model with core region as Heschel-Bulkely fluid and outlying region as Newtonian fluid described the mathematical model for catheterized artery [6]. They continued their study by assuming the blood as a two-fluid model which core region as a non-Newtonian fluid and the outer layer as a Newtonian fluid. The non-Newtonian fluid in the core region of the artery was represented by (i) Casson fluid and (ii) Herschel-Bulkley fluid. The expressions for the flow quantities obtained by Sankar [7] for the two-fluid Casson model and Sankar and Lee [8] for the two-fluid Herschel-Bulkley model were used to get the data for comparison. A catheterized artery with a clot inside the catheter was studied by Jayaraman and Dash [9]. The model follows that presented by Doffin and Chagneau [10]. Fluid dynamic parameters like wall shear stress, pressure drop and streamline patterns were discussed. Mekheimer and Kot [11] analyzed a catheterized artery in a balloon in the catheter. That study modeled blood for the axi-symmetric flow within co-axial tubes and outlying tube with an axially non-symmetric inner tube (catheter) include a balloon and estimated the effect of the stenosis shape and catheter with a balloon. Biswas and Chakraborty [12] considered a tapered position in addition to the stenosis artery and considered it for a steady blood flow. Blood fluid was assumed Newtonian fluid in co-axial circular cylinders. A velocity slip condition was employed at the catheterized wall with dissimilar sizes of stenosis. The effects of tapering and stenosis were considered. Daripa and Dash [13] assumed the catheter has eccentric position in an artery and studied the pulsatile blood flow in it. That investigation was carried out numerically by making use of an extended version of the fast algorithm. The mathematical model involved the common hypothesis that the arterial part was straight, blood was a Newtonian fluid. The axial pressure gradient and velocity distribution in the eccentric catheterized artery was obtained numerically. Mekheimer and Kot [14] studied blood flow between two eccentric tubes where the inner tube represents catheter while the outer tube was a tapered artery with stenosis. Blood flow was analyzed mathematically by considering it as a Newtonian fluid for a mild stenotic artery.

III. PROPOSED MODEL DEVELOPMENT

The objective of this step is to develop a mathematical model, solve it algebraically and finally, validation of predicted results with the published experimental data for an eccentric catheter through stenosis artery. Fig. 1 illustrates the schematic of the eccentric catheter through stenosis artery employed in this research.

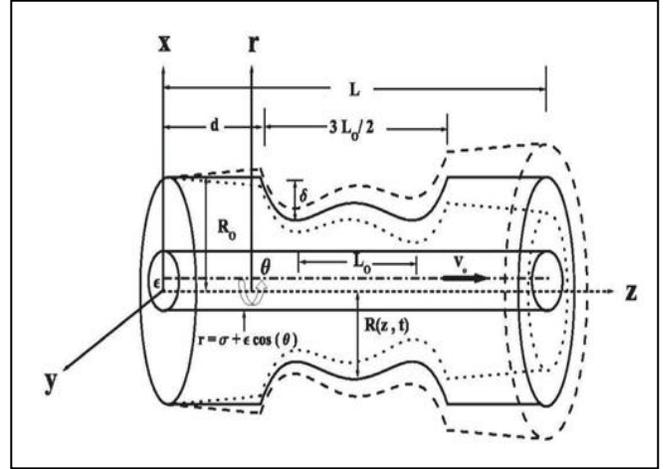


Figure 1. Schematic diagram of eccentric catheter through stenosed artery.

A. Model Assumptions

To simplify problem complexities, the following main assumptions were made in the present model:

- 1) *flow is as a Newtonian fluid*
- 2) *The whole process is in Steady-state condition.*
- 3) *Incompressible fluid.*
- 4) *Constant viscosity, other body forces (forces per unit volume), such as gravity or centrifugal force are not considered.*
- 5) *The temperature effects are also neglected.*

B. Model Equations

The two-dimensional model being presented in this study consists of non-linear, partial differential equations representing the conservation equations of mass, momentum, and species. The governing equations are summarized in Table I. According to our study area the governing equation should be written in cylindrical coordinate. These equations are also presented in Table II.

TABLE I. GOVERNING EQUATIONS

Type of equation	Formula	Eqs. numbers
Mass equation	$\nabla \cdot \tilde{v} = 0$	(1)
Momentum equation	$\frac{\partial \tilde{v}}{\partial t} + \nabla \cdot (\tilde{v}\tilde{v}) + \frac{\nabla p}{\rho} = +\mu \nabla^2 \tilde{v}$	(2)

TABLE II. GOVERNING EQUATIONS IN CYLINDRICAL COORDINATE

Type of equation	Formula	Eqs. numbers
Momentum equation in r direction	$\rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} - \frac{v^2}{r} + v_z \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left(\nabla^2 v - \frac{v}{r^2} - \frac{2}{r^2} \frac{\partial v}{\partial \theta} \right)$	(4)
Momentum equation in θ direction	$\left(\frac{\partial v_\theta}{\partial t} + v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \mu \left(\nabla^2 v_\theta - \frac{v_\theta}{r^2} + \frac{2}{r^2} \frac{\partial v_r}{\partial \theta} \right)$	(5)
Momentum equation in z direction	$\frac{\partial v_z}{\partial t} + v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z} = -\frac{\partial p}{\partial z} + \mu \nabla^2 v_z$	(6)

C. Model Boundary Conditions

The main boundary conditions of the model are in equation (7) and (8) as follow:

$$v_r = v_\theta = v_z = 0 \quad \text{on} \quad r = R(z, t) \quad (7)$$

$$v_r = v_\theta = 0, \quad v_z = v_0 \quad \text{on} \quad r = \sigma + \varepsilon \cos \theta \quad (8)$$

It means that velocities of components in wall of catheter are zero (No-slip condition). Finally, all the equations described above were solved using perturbation method. We formulated by adding a "small" term to the mathematical description of the exactly solvable problem as name eccentricity parameter.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we try to model our approach respecting three QoSs: *Axial Velocity*, *Wall Shear Stress* and *Resistance Impedance*.

A. Axial Velocity

In Fig. 2 (a), the variation in velocity in radial direction is shown for different values of radius of catheter by fixing the other parameters. We can see that, the axial velocity decreases by increasing the radius of catheter radius of catheter near the wall of catheter but near the wall of stenosis, the axial velocity is independent approximately to radius of catheter. In addition, as expected at concentric catheter, the velocity value of the basic flow started at the radius of the catheter, where $\sigma = 0.1$. Hence, the basic flow is higher for eccentric catheter than that for concentric one. In Fig. 2 (b), the change of velocity in radial direction is shown for different values of eccentricity parameter by fixing the other parameters. We can see that, the axial velocity increases by increasing the eccentricity parameter near the wall of catheter. In Fig. 2 (c) the axial velocity for

various values of velocity of catheter [15] is shown. This means the velocity of catheter depends on the radius of catheter. This is in well agreement with the medical situation of arterial catheterization. It is indicated that the axial velocity (the basic flow) will increase as the velocity of catheter increases. In addition, as expected at $\varepsilon = 0, v_0 = 0$, the value of axial velocity is equal to zero at the radius of the catheter, where $\sigma = 0.1$. It is observed that due to the eccentricity effect, the flow is slower when $v_0 = 0$, however, when v_0 is large the basic flow manages to overtake the latter, these results closely with those of [15] by letting $\varepsilon = 0$. In addition, the transmission of axial velocity through a moving catheter $v_0 \neq 0$ is substantially higher than that through a steady catheter $v_0 = 0$. Fig. 2 (d) shows the effect of the angle of circumferential direction on the basic flow, we can record that the axial velocity decreases as the angle of circumferential direction increases. Clearly, the relative importance of eccentricity is dependent on circumferential angle. For example, when $\theta = 90^\circ$, the velocity value of the basic flow started at the radius of the catheter, where $\sigma = 0.1$.

B. Wall Shear Stress

The wall shear stress is important in understanding the development of arterial disease because of the strong correlation between the localization of arteriosclerosis (stenosis) and arterial wall. The magnitude of the wall shear stress curve in the stenosis region ($0.75 \leq z \leq 2.25$) starts decreasing at the location of stenosis ($z = 0.75$) until its first maximum constriction ($z = 1$). Then it increases steeply to reach the middle point of the constriction ($z = 1.5$) where at the middle point the curve repeat its form again until its second maximum constriction ($z = 2$) where the distance between two maximum constrictions is equal to one $I_0 = 1$. Finally the curve increases gradually to reach the end of the constriction ($z = 2$). This is in well agreement with the physical situation of overlapping stenosis. The variation in the wall shear stress distribution in the stenosis region for different values of the maximum height of stenosis and taper angle are shown in Figures 3 (a) and (b). It is observed that the wall shear stress distribution decreases by increasing the maximum height of stenosis also the magnitude of the wall shear stress distribution is higher in the case of no-stenosis (uniform artery) than that for the case of stenosis artery. The effect of vessel tapering together with the shape of stenosis on the blood flow characteristics seem to be equally important and hence deserve special attention. The tapering is a significant aspect of mammalian arterial system. Thus, this research is interested in the flow through a tapered artery with stenosis. In an actual situation, the arterial wall thickness has not uniform shape, so we show the influences of the diverging, converging and non-tapering arterial on the wall shear stress.

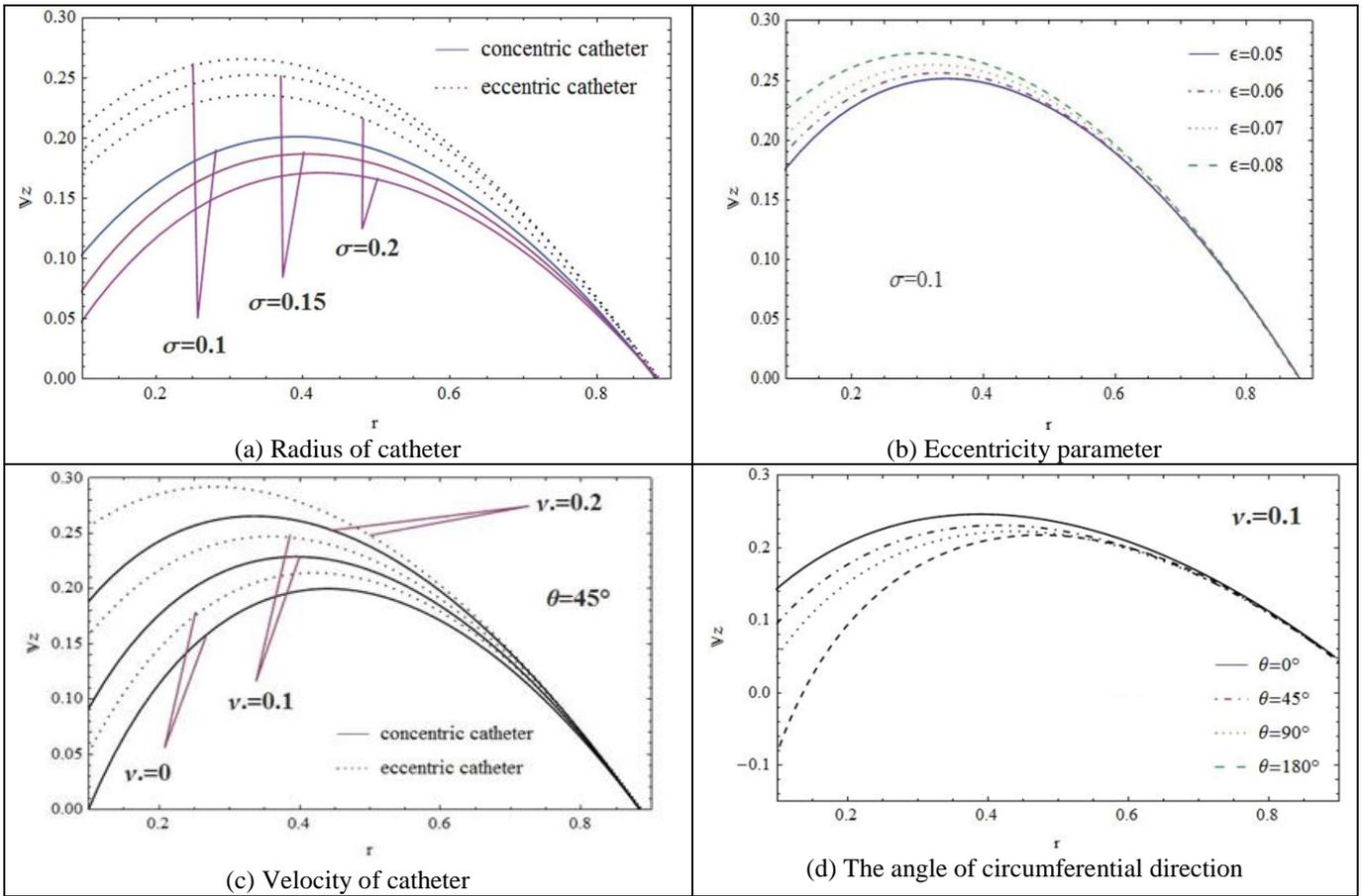
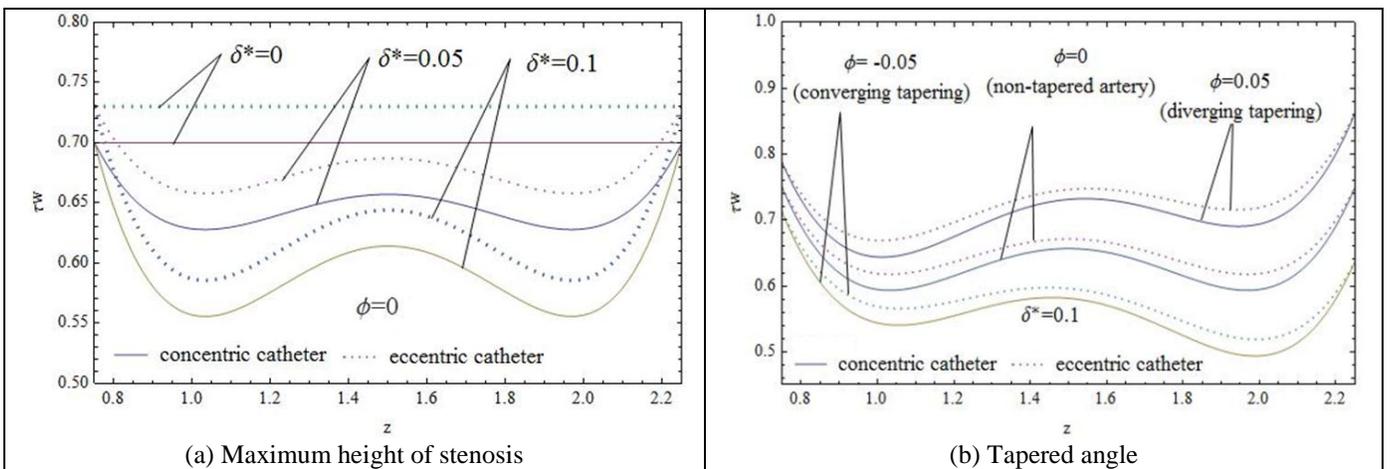


Figure 2. Change in axial velocity by variation in different parameters. (a) Radius of catheter, (b) Eccentricity parameter, (c) Velocity of catheter, (d) The angle of circumferential direction.



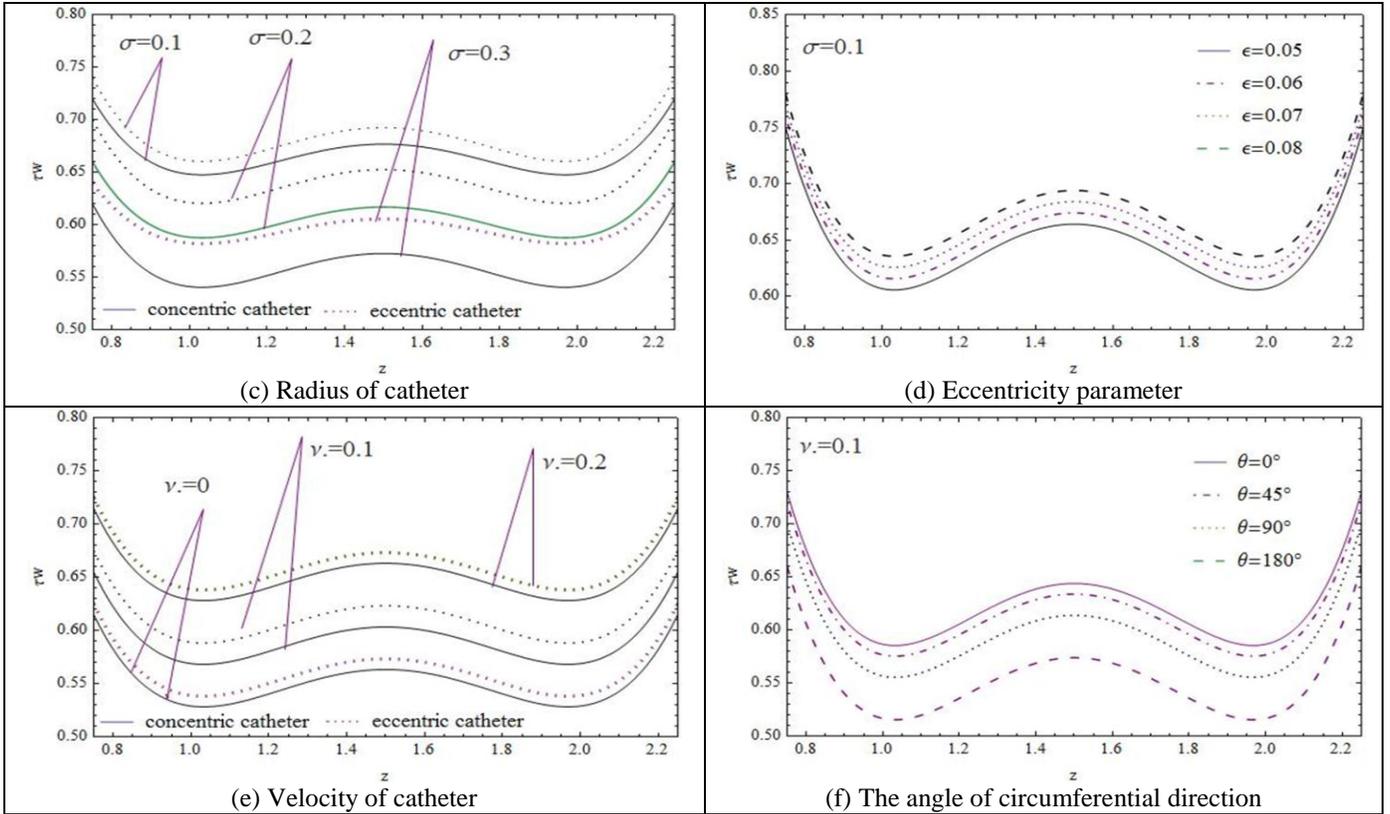


Figure 3. Change in wall shear stress with different parameters variation. (a) Maximum height of stenosis, (b) Tapered angle, (c) Radius of catheter, (d) Eccentricity parameter, (e) Velocity of catheter, (f) The angle of circumferential direction.

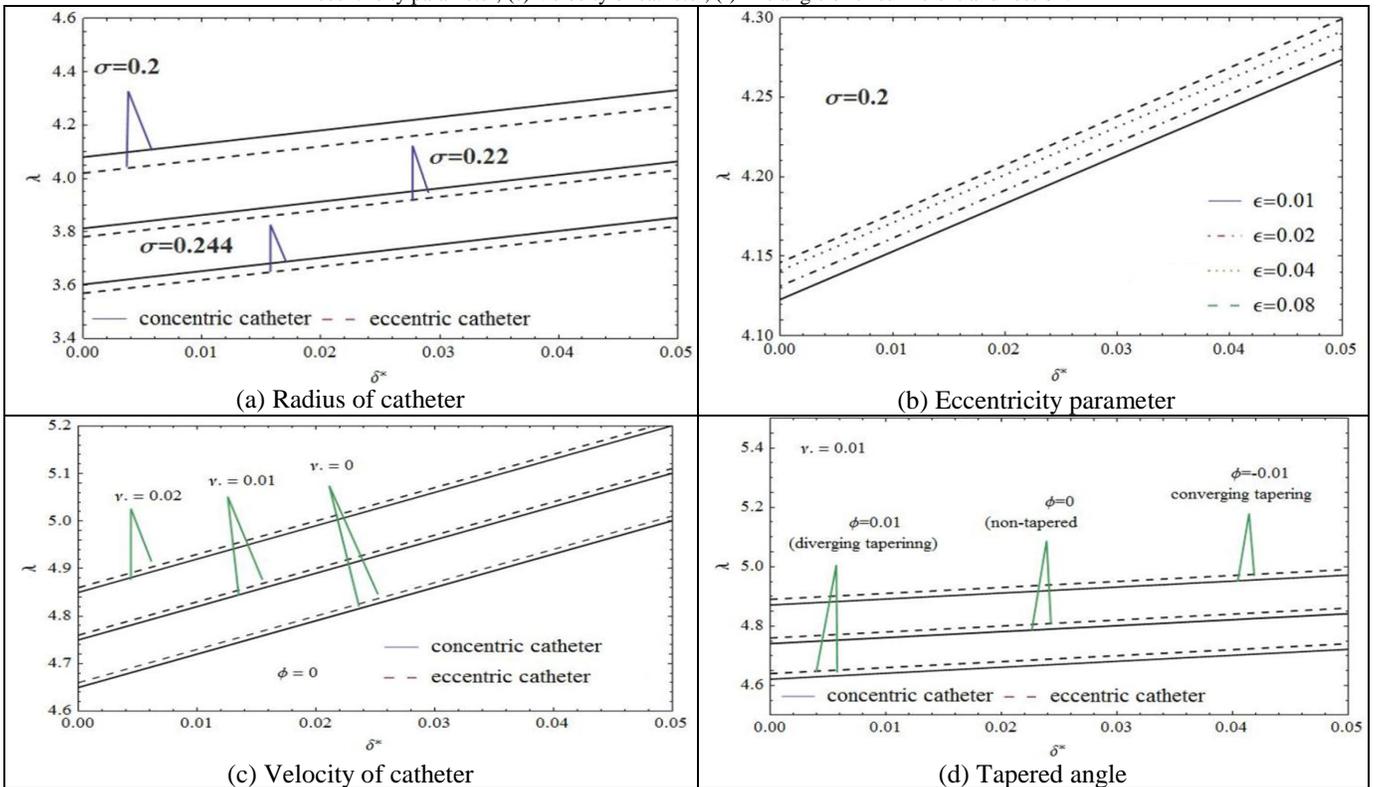


Figure 4. Change in resistance impedance variation with different parameters variation. (a) Radius of catheter, (b) Eccentricity parameter, (c) Velocity of catheter, (d) Tapered angle

It is clear that the curves through the diverging positive tapered artery are higher than in the non-tapered artery situation and the converging negative tapered artery situation. Figs. 3 (c) and (d) describe the distribution of the wall shear stress in the stenosis region for different values of Radius of catheter and Eccentricity parameter. The graphical results of these figures indicate that the wall shear stress increases by increasing the eccentricity parameters while it decreases by increasing the radius of catheter. The effect of the eccentricity is clearly visible from the plots shown earlier where the values of wall shear stress in the stenosis region are higher for eccentric catheter than that for concentric one. Figs. (e) and (f) we can decided that the wall shear stress curves unlike the characteristics of axial velocity with respect to the velocity of catheter and the angle of circumferential direction where the magnitude of wall shear stress increases by increasing velocity of catheter while it decreases as the angle of circumferential direction increases. In addition, the transmission of wall shear stress curves through a moving catheter is substantially higher than that through a steady catheter.

C. Resistance Impedance

Fig. 4 (a) to (d) illustrate the variation of the resistance to flow (resistance impedance) with the maximum height of stenosis, the eccentricity parameter, the radius of catheter, velocity of catheter and the taper angle respectively. It is observed that the resistance impedance has the same effect of the wall shear stress on eccentricity parameter, radius of catheter and velocity of catheter while the resistance impedance gives the reverse trend of the wall shear stress with respect to taper angle where the values of resistance are higher for converging tapering artery $\phi = -0.01 (< 0)$ rather than both the diverging tapering artery $\phi = 0.01 (> 0)$ and non-tapered artery $\phi = 0$. Hence, we can conclude that more blood can flow freely through diverging vessel, which has less effect of resistance. Trapping represents an interesting phenomenon for the fluid flow.

V. CONCLUSIONS

In this research a two dimensional, isothermal model is developed to determine the effects of eccentric catheterization on blood flow characteristics in a tapered and stenosis artery. The results can be concluded in following: The axial velocity is higher for eccentric catheter than that for concentric one. The transmission of axial velocity through a moving catheter $v_0 \neq 0$ is substantially higher than that through a steady catheter $v_0 = 0$. The axial velocity near the wall of stenosis does not depend on the eccentricity parameter

and radius of catheter. The magnitude of the wall shear stress distribution is higher for eccentric catheter than the concentric one. The values of wall shear stress through the diverging tapered artery $\phi > 0$ are higher than in the non-tapered artery situation $\phi = 0$ and the converging tapered artery situation $\phi < 0$.

The resistance impedance gives the reverse trend of the wall shear stress with respect to the taper angle where the blood can flow freely through diverging vessel, which has less effect on resistance. However, it must be noted that the resistance impedance in the case of eccentric catheter is less than that of the concentric one when the radius of catheter is considered. The trapping appears near the wall of catheter and the trapped bolus increases in size as the radius of catheter increases. while there is trapping bolus in the core region (the region between the wall of stenosis and the wall of catheter) for small values of eccentricity parameter. Increasing eccentricity parameter results in the trapping appears nearer to the wall of stenosis.

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