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Total time minimization of fuzzy transportation problem

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Abstract. This paper proposes a procedure for solving total time minimization in fuzzy transportation problem where the transportation time, source and destination parameters have been expressed as exponential fuzzy numbers by the decision maker. An algorithm is developed to obtain the optimal solution as exponential fuzzy number, which enables the decision maker to obtain more informing results and wider knowledge on the problem under consideration. A numerical example is solved to check the validity of the proposed procedure.

Keywords: Total time minimization, Fuzzy transportation problem, Fuzzy number, Exponential membership function, Graded mean integration representation

1. Introduction

The time minimizing transportation problem is encountered in connection with transportation of perishable goods, with the delivery of emergency supplies fire services, ambulance services or when military units are to be sent from their basis to the fronts. The time minimization transportation problem also known as the bottleneck transportation problem has already been studied by Arora and Puri [1], Burkard et al. [5] Garfrinkle and Rao [6], Hammer [8], Iserman [10], Szwarc [18], Mathur and Puri [11] and H.L. Bhatia, Swarup Kanti, M.C. Puri [4] and others.

There are two types of problems regarding the transportation time [19]: (i) minimization of the total transportation time (linear function, as aggregate the products of transportation time and quantity) called minimization of 1st transportation time and (ii) minimization of the transportation time of the longest active transporting route (non-linear function) called minimization of 2nd transportation time or problem of Barasov [3]. For (ii), the total number of units on transportation operation with longest time is minimized in [8]. An important variant of the total transportation time problems is formulated and resolved in [12], which is also included in [20].

The transportation time of the longest active transportation route(s) in problems where all destinations

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don't have the same importance are analyzed as the three classes single criteria and multi-criteria problems of the transportation time. The corresponding algorithms are developed in case of problems with priority according to demands of the subset of the destinations [14. Some bicriteria transportation problems are shown too [2, 13]. Another typical transportation problems are exposed in [16] and a research directive in [7].

In this paper, time minimization transportation is considered under uncertain environment due to geographic features such as low road density. A 2009 USITC report mentions that trucks in Ghana travelling from Paga (on the northern border with Burkina Faso) to Tema (on the Gulf of Guinea) take two to four days under normal conditions, but a week or more delays an estimated 10–20 percent of trucks. From a business or investment perspective, a long, but certain transport time may be preferable to a (potentially) shorter but unpredictable transport time.

Usually the imprecision are expressed as fuzzy numbers. In this paper, the transportation time, supply and demand is expressed as exponential fuzzy numbers, which are unlimited range of fuzzy numbers; but the generalized fuzzy numbers have a fixed range. An algorithm is developed to obtain the optimal solution as exponential fuzzy number. A numerical example is also given. The rest of the paper is organized as follows. Sections 2 and 3 introduce the concept of exponential fuzzy number and the general arithmetic operators. Fuzzy Time Minimization Transportation problem formulation is presented in Section 4 followed by solutions in Section 5. Experimental illustrations are provided in Section 5 and some conclusions are provided towards the end.

2. Exponential fuzzy number

Definition 2.1. In general, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function μ_A satisfies the following conditions.

(1) μ_A is a continuous mapping from R to the closed interval [0, 1].
(2) $\mu_A(x) = 0, -\infty < x \leq C$,
(3) $\mu_A(x) = L(x)$ is strictly increasing on [C, a],
(4) $\mu_A(x) = w, a \leq x \leq b$,
(5) $\mu_A(x) = R(x)$ is strictly increasing on [b, d],
(6) $\mu_A(x) = 0, d \leq x < \infty$,

Where $0 < w \leq 1$, a, b, c and d are real numbers.

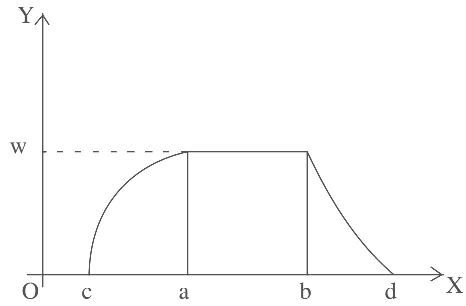

Fig. 1. Generalized LR type Fuzzy number $A = (c, a, b, d; w)_{LR}$.

We denote this type of generalized fuzzy number as $A = (c, a, b, d; w)_{LR}$ when $w = 1$, we denote this type of generalized fuzzy number as $A = (c, a, b, d)_{LR}$. Please see Fig. 1 for a illustration.

However these fuzzy numbers always have a fixed range as [c, d].

Definition 2.2. An exponential fuzzy number family is unlimited range fuzzy number. We define its general form as follows

$$f_A(x) = \begin{cases} w_A \exp\{-[(a_A - x)/\alpha_A]\}, & x \leq a_A \\ w_A, & a_A \leq x \leq b_A \\ w_A \exp\{-[(x - b_A)/\beta_A]\}, & b_A \leq x \end{cases}$$

where $0 < w_A \leq 1$, a_A, b_A are real numbers and α_A, β_A are positive real numbers. We denote this type of generalized exponential fuzzy number as $(A)_E = (a_A, b_A, \alpha_A, \beta_A; w_A)_E$. When $w_A = 1$, we denote it as $(A)_E = (a_A, b_A, \alpha_A, \beta_A)_E$. Please see Fig. 2 for a illustration.

Result 2.3. Let $(A)_E = (a_A, b_A, \alpha_A, \beta_A; w_A)_E$ be a generalized exponential number with $0 < w_A \leq 1$ and α_A, β_A are positive real numbers, a_A, b_A are real numbers then the graded mean integration representation of A is $P(A) = \frac{a_A + b_A}{2} + \frac{\beta_A - \alpha_A}{4}$

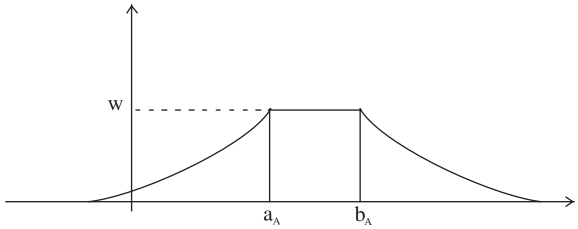

Fig. 2. Exponential membership function.

3. General arithmetic operators of exponential fuzzy numbers

Suppose that $(A_1)_E = (a_1, b_1, \alpha_1, \beta_1, w_1)_E$ and $(A_2)_E = (a_2, b_2, \alpha_2, \beta_2, w_2)_E$ are two generalized exponential fuzzy numbers. Let $w = \min\{w_1, w_2\}$ Some arithmetical results could be defined as:

1. The addition of $(A_1)_E$ and $(A_2)_E$ is

$$(A_1 \oplus A_2)_E = (a_1 + a_2, b_1 + b_2, \alpha_1 + \alpha_1, \beta_1 + \beta_2; w)_E$$

   Where a_1, b_1, a_2, b_2 are all real numbers and $\alpha_1, \alpha_2, \beta_1, \beta_2$ are positive.
2. The multiplication of $(A_1)_E$ and $(A_2)_E$ i

$$(A_1 \otimes A_2)_E = (a, b, \alpha, \beta, w)_E$$

   where $T = \{a_1a_2, a_1b_2, b_1b_2\}$, $T = \{\alpha_1\alpha_2, \alpha_1\beta_2, \beta_1\beta_2\}$
   and a = min T = kth element of T,

$$b = \max T = \text{lth element of } T,$$

   Then $\alpha = \min T_1$ = kth element of T_1,

$$\beta = \max T_1 = \text{lth element of } T_1,$$

$$\text{where } 1 \le K \le 4, 1 \le l \le 4.$$

3. $(-A_2)_E = (-b_2, -a_2, b_2, a_2; w_2)_E$ then

$$\begin{aligned}(A_1 \ominus A_2)_E &= (A_1 \oplus (-A_2))_E \\ &= (a_1 - b_2, b_1 - a_2, \alpha_1 + \beta_2, \beta_1 + \alpha_2; w)_E\end{aligned}$$

4. $\left(\frac{1}{A_2}\right)_E = \left(\frac{1}{b_2}, \frac{1}{a_2}, \frac{1}{\beta_2}, \frac{1}{\alpha_2}; w_2\right)_E$

$$\text{We have } \left(\frac{A_1}{A_2}\right)_E = \left(A_1 \otimes \left(\frac{1}{A_2}\right)\right)_E = \left(\frac{a_1}{b_2}, \frac{b_1}{a_2}, \frac{\alpha_1}{\beta_2}, \frac{\beta_1}{\alpha_2}; w\right)_E$$

   where $a_1\ b_1, a_2, b_2, \alpha_1, \alpha_2, \beta_1, \beta_2$ are all non-zero positive real numbers.
5. *Let* $m \in R^+$, $(A)_E = (a, b, \alpha, \beta; w)_E$, then
   $(m \otimes A)_E = (ma, mb, m, m\beta; w)_E$,
   If $m \in R^-$, $(A)_E = (a, b, \alpha, \beta; w)_E$, then
   $(m \otimes A)_E = (ma, mb, |m|\alpha, |m|\beta; w)_E$,

Definition 3.1. Let $(A_1)_E = (a_1, b_1, \alpha_1, \beta_1, w_1)_E$ and $(A_2)_E = (a_2, b_2, \alpha_2, \beta_2, w_2)_E$ be two generalized exponential fuzzy numbers, then $(A_1)_E > (A_2)_E$ if and only if $(P(A_1))_E > (P(A_2))_E$.

Definition 3.2. Let $(A_1)_E = (a_1, b_1, \alpha_1, \beta_1, w_1)_E$ and $(A_2)_E = (a_2, b_2, \alpha_2, \beta_2, w_2)_E$ be two generalized exponential fuzzy numbers, then $(A_1)_E = (A_2)_E$ if and only if $(P(A_1))_E = (P(A_2))_E$.

4. Formulation of the fuzzy time minimization transportation

Let us consider the standard balanced fuzzy transportation problem with m sources A_i (with supplies $(a_i)_E$), $i \in I = 1, 2, \ldots, m$) and n destinations B_j (with demands $(b_j)_E$), $j \in J = \{1, 2, \ldots, n\}$. If $(x_{ij})_E$ = the number of units moving from A_i to B_j, the feasible solution $(x)_E$ and the set of feasible solution $(X)_E$ is

$$(X)_E = \left\{ (x)_E \Big/ \sum_{j\in J} (x_{ij})_E = (a_i)_E, \forall\, i \in I; \sum_{i\in I} (x_{ij})_E = (b_j)_E, \forall\, j \in J; P(x_{ij})_E \ge 0, \forall (i, j) \right\} \quad (1)$$

$$\sum (a_i)_E = \sum (b_j)_E$$

i.e., $P(a_i)_E = P(b_j)_E$ for i to 1 to m, j = 1 to *n*.

Suppose $(t_{ij})_E$ = the time required for transporting all $(x_{ij})_E$ units using corresponding routes (i, j) for all $i \in I$ and $j \in J$.

Transportation efficiency $(F(x))_E$

$$= \sum_{i\in I} \sum_{j\in J} (t_{ij})_E (x_{ij})_E \quad (2)$$

Now we focus to the objective of minimizing the fuzzy time of active transportation routes (i, j) as

$$(T(x))_E = \sum_{i\in I} \sum_{j\in J} (t_{ij})_E h_{ij} \quad (3)$$

where h_{ij} as auxiliary function show active and non active transportation routes (activities)

$$h_{ij} = \begin{cases} 1, & \text{if } P(X_{ij})_E > 0 \\ 0, & \text{if } P(X_{ij})_E = 0 \end{cases} \quad (4)$$

This two types of measure of transportation efficiency (2) & (3) will be called Variant A (linear

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function) and Variant B (nonlinear function) of the total time transportation problem respectively.

5. Solution methodology

Let $X^{(k)}$ and $X^{(k+1)}$ are two basis neighbouring feasible solutions, where $(X_{ij}{}^{(k)})_E$ is entering basis variable and $(X_{is}{}^{(k)})_E$ is leaving basis variable for $X^{(k)}$.

$X^{(k)}$ contain: $P(X_{ij}{}^{(k)})_E = 0$ and $P(X_{is}{}^{(k)})_E > 0$

$X^{(k+1)}$ contain: $P(X_{ij}{}^{(k+1)})_E > 0$ and $P(X_{is}{}^{(k+1)})_E = 0$ there is $P(X_{ij}{}^{(k+1)})_E = P(X_{is}{}^{(k)})_E$.

In moving from $X^{(k)}$ to $X^{(k+1)}$, the fuzzy total time $(T(x))_E$ given as (3) will be changed with following values:

$$(q_{ij}^{(k)})_E = (t_{ij})_E - (t_{is})_E \tag{5}$$

The characteristic $(q_{ij})_E$ is the change of the transportation time in problem (3).

Then the solution $X^{(k+1)}$ has

$$(T^{(k+1)})_E = (T^{(k)})_E + (q_{ij}^{(k)})_E \tag{6}$$

Therefore the total time $(T^{(k+1)})_E$ is determined by values $(q_{ij}{}^{(k)})_E$ as follows:

$$P((T^{(k+1)})_E) = \begin{cases} >P(T^{(k)})_E & \text{if } P(q_{ij}^{(k)})_E > 0 \\ =P(T^{(k)})_E & \text{if } P(q_{ij}^{(k)})_E = 0 \\ <P(T^{(k)})_E & \text{if } P(q_{ij}^{(k)})_E < 0 \end{cases} \tag{7}$$

Let $(T^*)_E$ be the minimum value of $(T(x))_E$, $(X^*)_E$ is the fuzzy optimal solution of (3).

$$(T^*)_E = \min_X \left\{ (T(x))_E = \sum_{i\in I} \sum_{j\in J} (t_{ij})_E h_{ij} \right\} \tag{8}$$

The above discussion makes possible to develop the solving methods for defined transportation problem (3).

Also, the longest time on the separable active transportation routes is

$$(t(x))_E = \max_{P(x_{ij})_E > 0} (t_{ij})_E.$$

The following algorithm finds the fuzzy optimal solution and minimum fuzzy total transportation time (3).

5.1. Algorithm

Step 1: Find the fuzzy initial basic feasible solution $(X^{(1)})_E$ by Fuzzy Vogel's Approximation Method. Set the number of iteration $K = 1$.

Step 2: Determine the indicators $h_{ij}^{(k)}$ of active transportation routes $P(x_{ij}^{(k)})_E > 0$ and the total time $(T^{(k)})_E = T(x^{(k)})_E$.

$$h_{ij} = \begin{cases} 1, & \text{if } P(x_{ij}^{(k)})_E > 0 \\ 0, & \text{if } P(x_{ij}^{(k)})_E = 0 \end{cases} \tag{9}$$

$$\left(T^{(k)}\right)_E = \sum_{i\in I} \sum_{j\in J} (t_{ij})_E h_{ij}^{(k)} \tag{10}$$

Step 3: Determine the characteristics $(q_{ij}^{(k)})_E$ for all non-basic variables $x_{ij}^{(k)}$ using (5). Use the changing path of the basic solution (as in Stepping-Stone method) and corresponding leaving basic variable, eg. $(x_{is}^{(k)})_E > 0$ becomes $(x_{is}^{(k+1)})_E = 0$, if entering basic variable would be $(x_{ij}^{(k+1)})_E > 0$.

Step 4: Check the optimality of total fuzzy time (3) using (7). If all $P(q_{ij}^{(k)})_E \geq 0$ the optimal fuzzy solution $(X^*)_E$ is found. Stop the procedure. Otherwise, go to Step 5.

Step 5: Determine the next basic solution using entering variable $(x_{ij})_E$ with minimum $(q_{ij}^{(k)})_E$, regarding $P(q_{ij}^{(k)})_E < 0$. Set $K = K + 1$ and go to step 2.

6. Experimental illustration

Let us consider the following fuzzy time minimization transportation problem with m = 4 sources A_i, $i \in I = \{1, 2, 3, 4\}$ and $n = 5$ destinations B_j, $j \in J = \{1, 2, 3, 4, 5\}$. The transportation time, supply and demand which are expressed as exponential fuzzy numbers are presented in Table 1. Each row corresponds to a supply point and each column to a demand point. The total supply $(61, 69, 65, 75, 0.7)_E$ is equal to the total demand (60, 70, 65, 75, 0.8). In each cell (i, j) top left corner represents the time $(t_{ij})_E$ required for transporting $(x_{ij})_E$ units from source Ai to destination Bj. The basic variables $(x_{ij})_E$ are presented in the middle of corresponding cells (Bold letters) and the increase $(q_{ij})_E$ of time in bottom right corner of each cell (i, j) for the non-basic variable.

$$\sum (A_i)_E = (61, 69, 65, 75; 0.7)_E$$

$$\sum (B_i)_E = (60, 70, 65, 75; 0.8)_E$$

$$P(\sum (A_i)_E) = P(\sum (B_i)_E) = 67.5$$

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Table 1
The transportation time, supply and demand expressed as exponential fuzzy numbers

Sources/ destinations	B_1	B_2	B_3	B_4	B_5	Supply, a_i
A_1	$(10, 12, 11, 14; 0.7)_E$	$(2, 4, 3, 5; 0.8)_E$	$(9, 11, 10, 12; 0.8)_E$	$(1, 3, 2, 5; 0.7)_E$	$(4, 6, 5, 7; 0.8)_E$	$(13, 15, 14, 17; 0.7)_E$
A_2	$(1, 3, 2, 5; 0.7)_E$	$(6, 8, 7, 9; 0.8)_E$	$(2, 4, 3, 5; 0.8)_E$	$(7, 9, 8, 11; 0.7)_E$	$(1, 1, 2, 3; 0.9)_E$	$(12, 14, 13, 15; 0.8)_E$
A_3	$(11, 13, 12, 14; 0.8)_E$	$(1, 3, 2, 5; 0.7)_E$	$(3, 5, 4, 6; 0.8)_E$	$(4, 6, 5, 7; 0.8)_E$	$(6, 8, 7, 9; 0.8)_E$	$(21, 23, 22, 25; 0.7)_E$
A_4	$(8, 10, 9, 11; 0.8)_E$	$(3, 5, 4, 6; 0.8)_E$	$(5, 7, 6, 9; 0.7)_E$	$(2, 4, 3, 5; 0.8)_E$	$(4, 6, 5, 7; 0.8)_E$	$(15, 17, 16, 18; 0.8)_E$
Demand, b_j	$(14, 16, 15, 17; 0.8)_E$	$(9, 11, 10, 12; 0.8)_E$	$(14, 16, 15, 17; 0.8)_E$	$(9, 11, 10, 12; 0.8)_E$	$(14, 16, 15, 17; 0.8)_E$	

Table 2
Solution obtained by fuzzy Vogel's Approximation method

Sources/ destinations	B_1	B_2	B_3	B_4	B_5	Supply, a_i
A_1	$(10, 12, 11, 14; 0.7)_E$ $(4, 8, 18, 19; 0.7)_E$	$(2, 4, 3, 5; 0.8)_E$ **$(0, 6, 50, 51; 0.7)_E$**	$(9, 11, 10, 12; 0.8)_E$ $(5, 9, 15, 15; 0.8)_E$	$(1, 3, 2, 5; 0.7)_E$ **$(9, 11, 10, 12; 0.8)_E$**	$(4, 6, 5, 7; 0.8)_E$ **$(-4, 6, 77, 77; 0.7)_E$**	$(13, 15, 14, 17; 0.7)_E$
A_2	$(1, 3, 2, 5; 0.7)_E$ **$(12, 14, 13, 15; 0.8)_E$**	$(6, 8, 7, 9; 0.8)_E$ $(2, 6, 12, 12; 0.8)_E$	$(2, 4, 3, 5; 0.8)_E$ $(-2, 2, 8, 8; 0.8)_E$	$(7, 9, 8, 11; 0.7)_E$ $(4, 8, 13, 13; 0.7)_E$	$(1, 1, 2, 3; 0.9)_E$ $(-2, 0, 7, 5; 0.7)_E$	$(12, 14, 13, 15; 0.8)_E$
A_3	$(11, 13, 12, 14; 0.8)_E$ $(5, 9, 19, 19; 0.8)_E$	$(1, 3, 2, 5; 0.7)_E$ **$(5, 9, 39, 40; 0.7)_E$**	$(3, 5, 4, 6; 0.8)_E$ **$(14, 16, 15, 17; 0.8)$**	$(4, 6, 5, 7; 0.8)_E$ $(1, 5, 10, 9; 0.7)_E$	$(6, 8, 7, 9; 0.8)_E$ $(0, 4, 14, 14; 0.8)_E$	$(21, 23, 22, 25; 0.7)_E$
A_4	$(8, 10, 9, 11; 0.8)_E$ **$(0, 4, 30, 30; 0.8)_E$**	$(3, 5, 4, 6; 0.8)_E$ $(-1, 3, 9, 9; 0.8)_E$	$(5, 7, 6, 9; 0.7)_E$ $(-1, 5, 11, 12; 0.7)_E$	$(2, 4, 3, 5; 0.8)_E$ $(-1, 3, 8, 7; 0.7)_E$	$(4, 6, 5, 7; 0.8)_E$ **$(11, 17, 46, 48; 0.7)_E$**	$(15, 17, 16, 18; 0.8)_E$
Demand, b_j	$(14, 16, 15, 17; 0.8)_E$	$(9, 11, 10, 12; 0.8)_E$	$(14, 16, 15, 17; 0.8)_E$	$(9, 11, 10, 12; 0.8)_E$	$(14, 16, 15, 17; 0.8)_E$	

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The initial basic feasible solution is obtained by fuzzy Vogel's Approximation method is presented in Table 2.

$$\begin{aligned}(F^{(1)})_E &= (2, 4, 3, 5; 0.8)_E \,.\, (0, 6, 50, 51; 0.7)_E + (1, 3, 2, 5; 0.7)_E \,.\, (9, 11, 10, 12; 0.8)_E \\ &\quad + (4, 6, 5, 7; 0.8)_E \,.\, (-4, 6, 77, 77; 0.7)_E + (1, 3, 2, 5; 0.7)_E \,.\, (12, 14, 13, 15; 0.8)_E \\ &\quad + (1, 3, 2, 5; 0.7)_E \,.\, (5, 9, 39, 40; 0.7)_E + (3, 5, 4, 6; 0.8)_E \,.\, (14, 16, 15, 17; 0.8)_E \\ &\quad + (8, 10, 9, 11; 0.8)_E \,.\, (0, 4, 30, 30; 0.8)_E + (4, 6, 5, 7; 0.8)_E \,.\, (11, 17, 46, 48; 0.7)_E \\ &= (88, 384, 1219, 1897; 0.7)_E\end{aligned}$$

$$\begin{aligned}(T^{(1)})_E &= (2, 4, 3, 5; 0.8)_E + (1, 3, 2, 5; 0.7)_E + (4, 6, 5, 7; 0.8)_E + (1, 3, 2, 5; 0.7)_E + (1, 3, 2, 5; 0.7)_E \\ &\quad + (3, 5, 4, 6; 0.8)_E + (8, 10, 9, 11; 0.8)_E + (4, 6, 5, 7; 0.8) \\ &= (24, 40, 32, 51; 0.7)_E\end{aligned}$$

$$\begin{aligned}(t^{(1)})_E &= \max\{(t_{12})_E, (t_{14})_E, (t_{15})_E, (t_{21})_E, (t_{32})_E, (t_{33})_E, (t_{41})_E, (t_{45})_E\} \\ &= (t_{41})_E \\ &= (8, 10, 9, 11; 0.8)_E\end{aligned}$$

The indicators of $(d_{ij}{}^{(1)})_E$ for $(F(x))_E$ the indicators $(q_{ij}{}^{(1)})_E$ are shown in Table 3.

Since all $P(d_{ij})_E \geq 0$, the optimal solution for $(F(x))_E$ is reached.

Since $(q_{25}{}^{(1)})_E = (-2, 0, 7, 5; 0.7)_E$, i.e., $p(q_{25}{}^{(1)})_E < 0$, $(X^{(1)})_E$ is not optimal fuzzy solution of $(T(x))_E$. Taking $(x_{25})_E$ as entering variable $(x_{21})_E$ as leaving variable decrease $(T^{(1)})_E = (24, 40, 32, 51; 0.7)_E$ to $(T^{(2)})_E = (24, 38, 32, 49; 0.7)_E$ which is shown in Table 4.

$$\begin{aligned}(T^{(2)})_E &= (2, 4, 3, 5; 0.8)_E + (1, 3, 2, 5; 0.7)_E \\ &\quad + (4, 6, 5, 7; 0.8)_E + (1, 1, 2, 3; 0.9)_E \\ &\quad + (1, 3, 2, 5; 0.7)_E + (3, 5, 4, 6; 0.8)_E \\ &\quad + (8, 10, 9, 11; 0.8)_E + (4, 6, 5, 7; 0.8)_E \\ &= (24, 38, 32, 49; 0.7)_E\end{aligned}$$

Table 3
Various indicators

Non basic cell	Indicators $d_{ij}^{(1)}$ for F(x)	Indicators $q_{ij}^{(1)}$ for T(x)
$(x_{11}^{(1)})_E$	$d_{11}^{(1)} = (-2, 6, 34, 35, 0.7)_E$	$q_{11}^{(1)} = (4, 8, 18, 19, 0.7)_E$
$(x_{13}^{(1)})_E$	$d_{13}^{(1)} = (1, 9, 23, 24, 0.7)_E$	$q_{13}^{(1)} = (5, 9, 15, 15, 0.8)_E$
$(x_{22}^{(1)})_E$	$d_{22}^{(1)} = (5, 17, 38, 37, 0.7)_E$	$q_{22}^{(1)} = (2, 6, 12, 12, 0.8)_E$
$(x_{23}^{(1)})_E$	$d_{23}^{(1)} = (-3, 23, 42, 42, 0.7)_E$	$q_{23}^{(1)} = (-2, 2, 8, 8, 0.8)_E$
$(x_{24}^{(1)})_E$	$d_{24}^{(1)} = (7, 9, 39, 38, 0.7)_E$	$q_{24}^{(1)} = (4, 8, 13, 13, 0.7)_E$
$(x_{25}^{(1)})_E$	$d_{25}^{(1)} = (0, 6, 23, 21, 0.7)_E$	$q_{25}^{(1)} = (-2, 0, 7, 5, 0.7)_E$
$(x_{31}^{(1)})_E$	$d_{31}^{(1)} = (2, 8, 43, 42, 0.7)_E$	$q_{31}^{(1)} = (5, 9, 19, 19, 0.8)_E$
$(x_{34}^{(1)})_E$	$d_{34}^{(1)} = (0, 8, 18, 16, 0.7)_E$	$q_{34}^{(1)} = (1, 5, 10, 9, 0.7)_E$
$(x_{35}^{(1)})_E$	$d_{35}^{(1)} = (-1, 7, 22, 21, 0.7)_E$	$q_{35}^{(1)} = (0, 4, 14, 14, 0.8)_E$
$(x_{42}^{(1)})_E$	$d_{42}^{(1)} = (-3, 5, 21, 21, 0.8)_E$	$q_{42}^{(1)} = (-1, 3, 9, 9, 0.8)_E$
$(x_{43}^{(1)})_E$	$d_{43}^{(1)} = (-5, 7, 34, 33, 0.7)_E$	$q_{43}^{(1)} = (1, 5, 11, 12, 0.7)_E$
$(x_{44}^{(1)})_E$	$d_{44}^{(1)} = (-3, 5, 17, 20, 0.7)_E$	$q_{44}^{(1)} = (-1, 3, 8, 7, 0.7)_E$

Table 4
Quality of solutions

Sources/ destinations	B_1	B_2	B_3	B_4	B_5
A_1	$(10, 12, 11, 14; 0.7)_E$ $(4, 8, 18, 19; 0.7)_E$	$(2, 4, 3, 5; 0.8)_E$ **$(0, 6, 50, 51; 0.7)_E$**	$(9, 11, 10, 12; 0.8)_E$ $(5, 9, 15, 15; 0.8)_E$	$(1, 3, 2, 5; 0.7)_E$ **$(9, 11, 10, 12; 0.8)_E$**	$(4, 6, 5, 7; 0.8)_E$ **$(-4, 6, 77, 77; 0.7)$**
A_2	$(1, 3, 2, 5; 0.7)_E$ $(0, 2, 5, 7; 0.7)_E$	$(6, 8, 7, 9; 0.8)$ $(2, 6, 12, 12; 0.8)_E$	$(2, 4, 3, 5; 0.8)$ $(-2, 2, 8, 8; 0.8)_E$	$(7, 9, 8, 11; 0.7)_E$ $(4, 8, 13, 13; 0.7)_E$	$(1, 1, 2, 3; 0.9)_E$ **$(12, 14, 13, 15; 0.8)_E$**
A_3	$(11, 13, 12, 14; 0.8)_E$ $(5, 9, 19, 19, 0.8)_E$	$(1, 3, 2, 5; 0.7)_E$ **$(5, 9, 39, 40, 0.7)$**	$(3, 5, 4, 6; 0.8)_E$ **$(14, 16, 15, 17, 0.8)$**	$(4, 6, 5, 7; 0.8)_E$ $(1, 5, 10, 9, 0.7)_E$	$(6, 8, 7, 9; 0.8)_E$ $(0, 4, 14, 14, 0.8)_E$
A_4	$(8, 10, 9, 11; 0.8)_E$ **$(12, 18, 43, 45; 0.8)_E$**	$(3, 5, 4, 6; 0.8)_E$ $(-1, 3, 9, 9; 0.8)_E$	$(5, 7, 6, 9; 0.7)_E$ $(-1, 3, 13, 14; 0.7)_E$	$(2, 4, 3, 5; 0.8)_E$ $(-4, 0, 10, 10; 0.8)_E$	$(4, 6, 5, 7; 0.8)_E$ **$(-3, 5, 61, 61; 0.7)_E$**

Table 5
Quality of solutions

Sources/ destinations	B_1	B_2	B_3	B_4	B_5
A_1	$(10, 12, 11, 14; 0.7)_E$ $(7, 11, 16, 16; 0.7)_E$	$(2, 4, 3, 5; 0.8)_E$ **$(0, 6, 50, 51; 0.7)_E$**	$(9, 11, 10, 12; 0.8)_E$ $(5, 9, 15, 15; 0.8)_E$	$(1, 3, 2, 5; 0.7)_E$ **$(4, 14, 71, 73; 0.7)_E$**	$(4, 6, 5, 7; 0.8)_E$ **$(-7, 11, 137, 137; 0.7)_E$**
A_2	$(1, 3, 2, 5; 0.7)_E$ $(-2, 2, 7, 7, 0.7)_E$	$(6, 8, 7, 9; 0.8)$ $(2, 6, 12, 12, 0.8)_E$	$(2, 4, 3, 5; 0.8)$ $(-2, 2, 8, 8, 0.8)_E$	$(7, 9, 8, 11; 0.7)$ $(4, 8, 13, 13, 0.7)_E$	$(1, 1, 2, 3; 0.9)_E$ **$(12, 14, 13, 15, 0.8)_E$**
A_3	$(11, 13, 12, 14; 0.8)_E$ $(8, 12, 17, 16; 0.7)_E$	$(1, 3, 2, 5; 0.7)_E$ **$(5, 9, 39, 40; 0.7)_E$**	$(3, 5, 4, 6; 0.8)_E$ **$(14, 16, 15, 17; 0.8)$**	$(4, 6, 5, 7; 0.8)_E$ $(1, 5, 10, 9; 0.7)_E$	$(6, 8, 7, 9; 0.8)_E$ $(0, 4, 14, 14; 0.8)_E$
A_4	$(8, 10, 9, 11; 0.8)_E$ **$(12, 18, 43, 45; 0.8)_E$**	$(3, 5, 4, 6; 0.8)_E$ $(-1, 3, 9, 9; 0.8)_E$	$(5, 7, 6, 9; 0.7)_E$ $(1, 5, 11, 12; 0.7)_E$	$(2, 4, 3, 5; 0.8)_E$ **$(-3, 5, 61, 61; 0.7)_E$**	$(4, 6, 5, 7; 0.8)_E$ $(0, 4, 10, 10; 0.8)_E$

$$\begin{aligned}(t^{(2)})_E &= \max\{(t_{12})_E, (t_{14})_E, (t_{15})_E, (t_{21})_E, (t_{32})_E,\\ &\quad (t_{33})_E, (t_{41})_E, (t_{45})_E\}\\ &= (t_{41})_E\\ &= (8, 10, 9, 11; 0.8)_E\end{aligned}$$

Since $(q_{44}^{(2)})_E = (-4, 0, 10, 10, 0.8)_E$, i.e., $P((q_{44}^{(2)})_E) < 0$ $(X^{(2)})_E$ is not optimal fuzzy solution of $(T(x))_E$. Taking $(x_{44})_E$ as entering variable $(x_{45})_E$ as leaving variable decrease $(T^{(2)})_E = (24, 38, 32, 49, 0.7)_E$ to $(T^{(3)})_E = (22, 36, 32, 47, 0.7)_E$ which is shown in Table 5.

Since all $P(q_{ij})_E \geq 0$, the optimal solution of $(T(x))_E$ is reached.

$$\begin{aligned}(T^{(3)})_E &= (2, 4, 3, 5; 0.8)_E + (1, 3, 2, 5; 0.7)_E\\ &\quad + (4, 6, 5, 7; 0.8)_E + (1, 1, 2, 3; 0.9)_E\\ &\quad + (1, 3, 2, 5; 0.7)_E + (3, 5, 4, 6; 0.8)_E\\ &\quad + (8, 10, 9, 11; 0.8)_E + (2, 4, 3, 5, 0.8)_E\\ &= (22, 36, 32, 47; 0.7)_E\end{aligned}$$

$$\begin{aligned}(t^{(3)})_E &= \max\{(t_{12})_E, (t_{14})_E, (t_{15})_E, (t_{21})_E, (t_{32})_E,\\ &\quad (t_{33})_E, (t_{41})_E, (t_{45})_E\}\\ &= (t_{41})_E\\ &= (8, 10, 9, 11; 0.8)_E\end{aligned}$$

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7. Conclusions

The time of transport might be significant factor in several transportation problems. In this paper, an algorithm is developed to solve total time minimization in fuzzy transportation problem. Optimal solution is obtained as exponential fuzzy number. This enables the decision maker to obtain more informing results and wider knowledge on the problem under consideration. Using the proposed algorithm, it is found that:

$$(T^{(1)})_E = (24, 40, 32, 51, 0.7)_E \quad P((T^{(1)})_E = 36.75$$

$$(T^{(2)})_E = (24, 38, 32, 49, 0.7)_E \quad P((T^{(2)})_E = 35.25$$

$$(T^{(3)})_E = (22, 36, 32, 47, 0.7)_E \quad P((T^{(3)})_E = 32.75$$

Which shows the efficiency of the proposed algorithm. In this paper, the input as well as the output is expressed as exponential fuzzy numbers, so that the proposed procedure preserves the fuzziness. The transportation efficiency and the longest time on active transportation routes are also obtained.

References

[1] S. Arora and M.C. Puri, On lexigraphical optimal solutions in transportation, *Management Science* **25** (1979), 73–78.

[2] Y.P. Aneya and K.P.K. Nair, Bicriteria transportation problem, *Management Science* **5** (1979), 73–78.

[3] A.S. Barasov, Linear Programming, Fizmatgiz, Moskva, 1961 (in Russian).

[4] H.L. Bhatia and Swarup Kanti, M.C. Puri, Time-cost trade-off in a transportation problem, *Opsearch* **13**(3–4) (1976), 129–142.

[5] R.E. Burkard and F. Rendl, Lexicographic bottleneck problems, *Operation Research Letters* **10** (1991), 303–308.

[6] R.S. Garfinkel and M.R. Rao, The Bottleneck transportation problem, *Naval Research Logistics Quarterly* **18** (1971), 465–472.

[7] L.F. Gomez and A.M., Multicriteria interactive programming for optimizing transportation Networks, A research directive, *Revista de transporte e, Technologia* **7**(14) (1995), 86–94.

[8] P.L. Hammer, Time minimizing transportation problems, *Naval Research Logistics Quarterly* **16** (1969), 345–357.

[9] Ilija Nikolic, Total time minimizing transportation problem, *Yugoslav Journal of Operation Research,* **17** (2007), 125–133.

[10] H. Isermann, Linear bottleneck transportation problem, *Asia – pacific Journal of Operation Research* **1** (1984), 38–52.

[11] K. Mathur and M.C. Puri, A bi-Level bottleneck programming problem, *European Journal of Operation Research* **86** (1955), 337–344.

[12] I. Nikolic, Minimization of total transportation time, SYM-OP-Is'86, Hereg-Novi, Yugoslavia, 1986, 519–520 (in Serbian).

[13] I. Nikolic, Bicriteria transportation total cost-total time problem, *SYMOP- IS'93*, (1993), 387–390 (in Serbian).

[14] I. Nikolic and J. Petric, Minimization of transportation time-multicriteria approach, Third mini EURO conference-operation research methods in transport planning and traffic control, Herceg Novi, Yugoslavia, 1987, 1–12.

[15] Shan-Huo Chen, Guo-Chin Li, Representation, ranking and distance of fuzzy number with expoential membership function using graded mean integration method, *Tamsui Oxford Journal of Mathematical Sciences* **16**(2) (2000), 123–131.

[16] P. Singh and P.K. Saxena, The multiple objective time transportation problem with Additional restrictions, *European journal of Operational Research* **146**(3) (2003), 460–476.

[17] D. Stephen Dinagar and K. Palanivel, On Trapezoidal membership function in solving transportation problem under fuzzy environment, *International Journal of computational physical sciences* **1**(1) (2009), 1–12.

[18] W. Szwarc, The time transportation problem, *Zastowania Matematyki (Warsaw)* **8** (1966), 231–242.

[19] S. Vukadinovic, Transportation problem in linear programming, Naucna knjiga Belgrade, Yugoslavia, 1985.

[20] S. Vukadinovic and S. Cvejic, Mathematical programming, University of Pristina, Yugoslavia, 1995 (in Serbian).

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