ON A CLASS OF GLOBAL OPTIMIZATION TEST FUNCTIONS

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Abstract: This paper provides a theoretical proof illustrating that for a certain class of functions having the property that the partial derivatives have the same equation with respect to all variables, the optimum value (minimum or maximum) takes place at a point where all the variables have the same value. This information will help the researchers working with high dimensional functions to minimize the computational burden due to the fact that the search has to be performed only with respect to one variable.

Key words: *Multidimensional functions, global optimization, first partial derivatives*

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1. Introduction

Among the global optimization test problems, there is a class of functions, which has not been treated until now separately by any of the conventional deterministic and stochastic approaches. In this context, we refer to the functions for which the first partial derivatives with respect to all variables have the same equation.

For example, let us consider the Rastrigin function given by:

$$F : [-5.12, 5.12]^n \to \Re$$

$$F(x) = 10n + \sum_{i=1}^{n}(x_i^2 - 10\cos(2\pi x_i)),$$

for which the global optimum is at (0,0,...,0).

In the two-dimensional case, the Rastrigin function is represented as:

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$$F(x,y) = 20 + x^2 - 10\cos(2\pi x) + y^2 - 10\cos(2\pi y).$$

The first partial derivatives are:

$$f(x) = \frac{\partial}{\partial x} F(x,y) = 2x + 20\pi \sin(2\pi x)$$

$$f(y) = \frac{\partial}{\partial y} F(x,y) = 2y + 20\pi \sin(2\pi y).$$

From the equations above, it is evident that both derivatives have the same equation, which is given by:

$$f(t) = 2t + 20\pi \sin(2\pi t).$$

Thus, for all other similar examples for which the first partial derivatives have the same equation(s), the global optimum of the function will be at a point where all the variables have the same value.

2. Problem Formulation and Proof

We attempt to formulate the problem and prove the following result:

Theorem 2.1 *Let $F : Q \to \Re$ be a twice-differentiable continuous function, where Q is a bounded and closed subset of $\Re^n$. If the first partial derivatives $f : Q \subset \Re^n \to \Re$ have the same equation with respect to all variables then F takes its global optimum (maximum or minimum) in a point where all the variables have the same value and where $g(t) = \int f(t)dt$ also takes its global optimum (maximum or minimum).*

Proof: Without loss of generality, we attempt to prove the theorem for three variables (for a better understanding) but a generalization is trivial.

Suppose we have $F = F(x,y,z)$ and all the three partial derivatives have the same equation $f(t)$.

Thus, from

$$\frac{\partial}{\partial x} F(x,y,z) = f(x) \tag{1}$$

we have:

$$F(x,y,z) = \int f(x)dx + \Phi(y,z). \tag{2}$$

From:

$$\frac{\partial}{\partial y} F(x,y,z) = f(y) \tag{3}$$

and (2) we obtain:

$$\frac{\partial}{\partial y}\left(\int f(x)dx + \Phi(y,z)\right) = \frac{\partial}{\partial y}\left(\Phi(y,z)\right) = f(y). \tag{4}$$

This implies:

$$\Phi(y,z) = \int f(y)dy + \Psi(z). \tag{5}$$

From:

$$\frac{\partial}{\partial z}F(x,y,z) = f(z) \tag{6}$$

(2) and (5) we obtain:

$$\frac{\partial}{\partial z}\left(\int f(x)dx + \int f(y)dy + \Psi(z)\right) = \frac{\partial}{\partial z}\Psi(z) = f(z). \tag{7}$$

This involves:

$$\Psi(z) = \int f(z)dz + c. \tag{8}$$

Thus from (2), (5) and (8) we obtain:

$$F(x,y,z) = \int f(x)dx + \int f(y)dy + \int f(z)dz + c, \tag{9}$$

where c is a constant.

Thus, to find the global minimum or maximum, we should take the same value for x, y, and z and these are also the values where f takes its global minimum or maximum. □

A graphical example for the sum of squared functions comprising of two variables is illustrated in Fig. 1. In the 2-dimensional case, this function is given by $F(x,y) = x^2 + y^2$. For this function, the optimum of F will be the same as the optimum of either x^2 or y^2. All the three functions (F, x^2, y^2) are depicted and presented from different angles. We considered the case of minimization of F within the range [-10, 10] for which the optimum point is at $t^* = (0,0)$ with $F(t^*) = 0$.

3. Perspectives and Usefulness

There is a huge amount of work dealing with global optimization. The results presented in this paper are addressed especially to researchers using various kinds of optimization techniques. On one hand, it is irrelevant to use these test functions and use immense computational efforts searching the solution space for each variable while, in fact, it is only required to search for one of them. On the other hand, the complexity of these approaches increases with the number of variables, which is one of the limitations why we are only able to deal with functions having only a few hundreds of variables. We believe that researchers are taking advantage of this property. In the literature, we could see only few works [12] dealing with some correlations among the variables, without explicitly mentioning this property

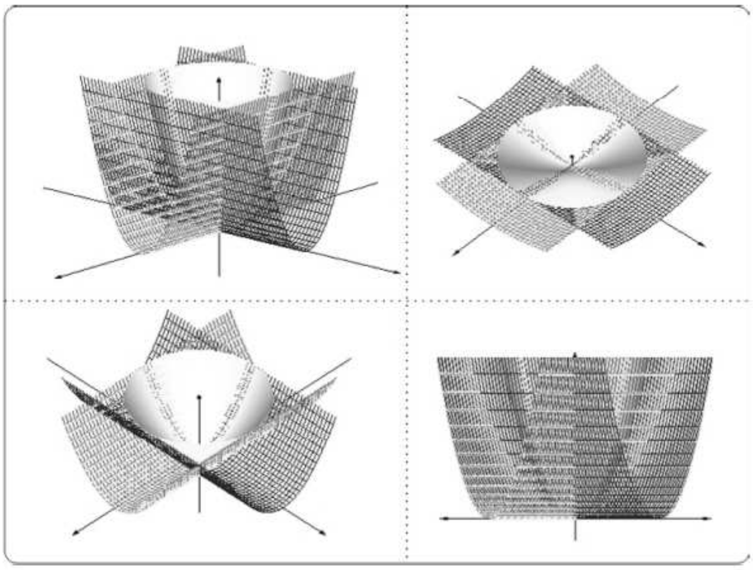

Fig. 1 *Example of initial function and function decomposition for the sum of squares function.*

or making use of it. In order to eliminate such correlations, there have been some attempts, like modification of the function itself by rotation, etc.

Floudas and Pardalos [3], Hedar and Fukushima [6] illustrated a list of the most used test functions. Not all the functions described have this property, but many of them and many others real-world problems do have it. As an example, some common functions are: Sum of Squares, Rastrigin, Levi, Quadric, Sphere function and many more.

The aim of this paper is to draw the optimization community's attention to the possibility of reducing the complexity in order to obtain faster and better results by considering some important properties of the functions as described above. The property of this type of functions should be exploited before an optimization algorithm is applied so that the complexity of the problem could be drastically reduced.

From the huge amount of work existing in the literature we can observe that the approaches encounter difficulties in dealing with high dimensional functions no matter what kind of technique is employed [1], [2], [4], [5], [7]–[11], [13]–[20]. The findings in this paper can help at least in dealing with practical applications which require fast results.

4. Discussions and Conclusions

This paper addresses a certain class of optimization functions which are twice continuously differentiable. It is observed and proved that the functions whose first partial derivatives have the same equation have an optimum at a point where all the variables have the same value. The authors' findings can help in approaching complicated test functions having this feature, which unfortunately continues to be ignored at least by a large research community employing various computationally intelligent optimization algorithms.

The class of functions where the assumption discussed in this paper is immediately seen is very special. For some of them this can be seen immediately (either the functions have less variables or they have a simple form). But, in general, this property is not difficult to observe by any mathematician.

Even though these findings, in essence, are very simple and absolutely trivial from a mathematical point of view, their advantages have never been exploited.

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