

Fuzzy Modeling System based on Hybrid Evolutionary Approach

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Abstract— In this paper, we introduce a new evolutionary methodology to design fuzzy inference systems. An innovative hybrid stages of learning method and tuning method, contains Subtractive clustering, Adaptive Neuro-Fuzzy Inference System (ANFIS) and particle swarm optimization (PSO), is developed to generate evolutionary fuzzy modeling systems with high accuracy. For the purpose of illustration and validation of the approach, some data sets have been exploited. Empirical results illustrate that the proposed method is efficient.

Keywords— *particle swarm optimization, Fuzzy Membership function, Fuzzy models, Adaptive Neuro-Fuzzy, Subtractive clustering.*

I. INTRODUCTION

Recently, computational Intelligence techniques such as fuzzy logic [1], artificial neural networks [2, 3] and evolutionary algorithms (EAs) [4-6] are becoming popular research subjects. They can deal complex problems which are difficult to be solved by classical techniques [7].

The concept of fuzzy modeling and fuzzy set proposed by Zadeh [8] has been widely investigated. Fuzzy systems are usually considered as one of the most important areas for the application of the Fuzzy Set Theory. They provide a scheme to represent the knowledge in a way that resembles human communication and reasoning. Fuzzy systems have demonstrated their ability in several application fields, such as control problems [9-11], classification [12-15], regression [16, 17], and general data mining problems [18, 19], due to their ability to handle uncertainty and imprecision and to describe the behavior of different complex systems without requiring a precise mathematical model.

Design of fuzzy model or fuzzy model identification is the task of finding the parameters of fuzzy models so as to get the desired behavior. In this case, the design of fuzzy models can be considered as an optimization task or a search problem. Thanks to their ability to find near-optimal solutions without a precise description of the problem, many intelligent optimization techniques have been employed to generate fuzzy models from numerical data and to tune the structure and the rules' parameter of the fuzzy systems. Among these intelligent techniques we can find clustering [19], artificial neural network [2, 3], evolutionary computation [20], and so on. In this context, this paper discusses a new approach of fuzzy

model identification problem making use of subtractive clustering, Adaptive Neuro-Fuzzy Inference System (ANFIS) and Particle Swarm Optimization (PSO) algorithm. The objective is to present the use of learning and tuning methods for building an optimal fuzzy model from the available data. The methodology presented in this work is carried out in two main steps: in the first one, structure learning is performed, i.e., a set of fuzzy rules is obtained; in the second one, the parameters of the model are tuned, i.e., the parameters of the membership functions of the fuzzy system. The strategy aims fundamentally at obtaining models with high prediction accuracy.

The paper is set up as follows. In Section 2 a brief introduction to fuzzy system modeling is introduced. Section 3 provides a brief account of PSO algorithm. Section 4 provides a description of the proposed hybrid methodology for fuzzy model identification. Simulation results considering different time series prediction problems are presented in section 5. Finally, conclusions are drawn in Section 6.

II. FUZZY SYSTEMS MODELING

Fuzzy modeling is the task of identifying the parameters of fuzzy inference system so as to achieve a desired behavior. The fuzzy model identification process involves the task of providing a methodology for development i.e. a set of techniques for obtaining the fuzzy model from information and knowledge about the system.

Generally, the problem of fuzzy model identification includes the following issues [21]:

- Selecting the type of fuzzy model.
- Selecting the input and output variables for the model.
- Identifying the structure of the fuzzy model, which includes determination of the number and types of membership functions for the input and output variables and the number of fuzzy rules.
- Identifying the parameters of antecedent and consequent membership functions.
- Identifying the consequent parameters of the fuzzy rule base.

These issues can be grouped into three sub-problems as shown in Figure 1: structure identification, parameter estimation and model validation. If the performance of the model obtained is not satisfactory, the model structure is modified and the parameters are re-estimated till the performance is satisfactory.

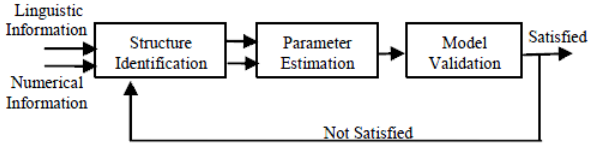


Fig. 1. Fuzzy Model Identification Process

III. PARTICLE SWARM OPTIMIZATION ALGORITHM

Particle Swarm Optimization (PSO) was proposed by Kennedy and Eberhart [22] as a population based stochastic optimization method inspired by the social behavior of bird flocking and fish schooling. PSO is a computationally effective algorithm based on group (swarm) behavior where each individual, referred to as a particle, represents a candidate solution. The individuals in the swarm cooperate. The algorithm searches for an optimal value by sharing social and cognitive information among the individuals (particles).

In each step, a particle moves to a new position by adjusting its velocity. The velocity is updated according to the global best position g_{best} and the best position of each particle p_{best} . The velocity and position update equations of the i -th particle in the swarm are given as follows:

$$V_i(t+1) = w * V_i(t) + C_1 * R_1 * (p_{best} - X_i(t)) + C_2 * R_2 * (g_{best} - X_i(t)) \quad (1)$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (2)$$

Where:

$V_i(t)$: Velocity of agent i at iteration t .

$X_i(t)$: Current position of agent i at iteration t .

C_1 and C_2 : cognitive/social acceleration.

w : inertia weight.

R_1 and R_2 : random numbers uniformly distributed in the range (0,1).

g_{best} : best position found by swarm (global best).

p_{best} : best position found by i -th particle (local best).

IV. THE PROPOSED METHODOLOGY FOR FUZZY MODELS IDENTIFICATION

When a model is developed based on the theory of system identification, its parameters are tuned according to some criteria, aiming to obtain a final representation, adequate for the modeling purposes. In this sense, a new hybrid methodology of learning and tuning methods is introduced. In spite of the adaptive ability of PSO algorithm, its training result is not desirable for the reason of incomplete learning cycles. For this reason and to well approximate the desired output, the input-output data is first clustered by means of subtractive clustering. At this point, we use the optimization capability of ANFIS to improve the model. Finally, PSO is utilized to quickly regulate adjustable parameters to construct the desired fuzzy modeling system. Therefore, the proposed approach fuzzy modeling system is enough to approach high accuracy within a short training time (Figure 2).

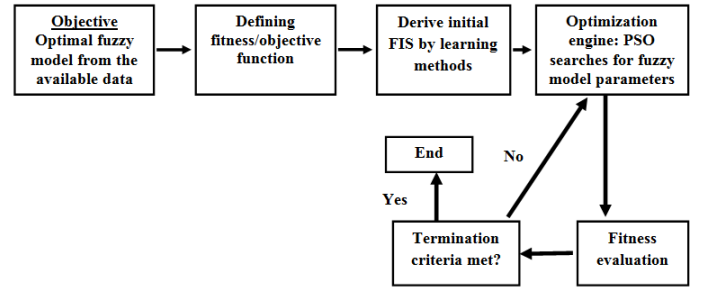


Fig. 2. Optimal fuzzy model identification using PSO as an optimization engine

A. Learning process

In this section, a learning process based on subtractive clustering and ANFIS will be introduced. For complex systems, fuzzy inference system based on only expert knowledge may suffer from a loss of accuracy. So, a learning process is applied. It relates to the task of directly obtaining the fuzzy rule surface [23] or deep structures [24] from the available data. As a result, the initial structure of the fuzzy system will be built up and we obtain a compact rule base with a reduced number of rules. Since the proposed tuning method does not reduce the rule base size, this fact fits well to the design approach.

The proposed learning process consists of two phases:

1) subtractive clustering

In the first phase, the structure of the model is obtained by means of subtractive clustering, which allows the extraction of a set of relevant rules based on a set of representative input-output data samples.

Clustering of numerical data forms the basis of many classification and system modeling algorithms. Cluster analysis is a technique that is used to seek out data, dividing all objects (samples) into smaller subgroups and classifying them according to the similarities among them. A fuzzy cluster is a fuzzy subset of the set of objects, with the membership function of each object representing the degree to which it belongs to that cluster [25]. We use the cluster information to generate a Sugeno-type fuzzy inference system that best models the data behavior using a minimum number of rules. The rules partition themselves according to the fuzzy qualities associated with each of the data clusters.

Subtractive clustering is a fast one-pass algorithm for estimating the number of clusters and the location of cluster centers in a set of data. The center candidates are the data samples themselves. After applying subtractive clustering, each of the obtained clusters will constitute a prototype for a particular behavior of the system under analysis. So, each cluster can be used to define a fuzzy rule capable of describing the behavior of the system in some region of the input-output space.

This technique not only makes determination of the number of clusters become simple, but also reduces the computational effort. In addition, an important advantage of using a clustering method to find rules is that the resultant rules are more tailored to the input data than they are in an FIS generated without clustering. This reduces the problem of combinatorial explosion of rules when the input data has a high dimension (the dreaded curse of dimensionality).

On the other hand, the cluster radius is an important parameter of the subtractive clustering algorithm. Specifying a small cluster radius will yield many small clusters in the data, (resulting in many rules). Specifying a large cluster radius will usually yield a few large clusters in the data, (resulting in fewer rules).

In our research, the number of cluster centers is equal to the number of the fuzzy rules. More details of Subtractive Clustering algorithm are presented in [26].

2) Adaptive Neuro-Fuzzy Inference System (ANFIS)

Subtractive clustering algorithm is used as a pre-processor to Adaptive Neuro-Fuzzy Inference System (ANFIS) for determining the initial rules. Then, in order to more improve the resulting Fuzzy Inference System (FIS), the parameters of the model are tuned via the training of a neural network through ANFIS. This phase applies an adaptive neural network to optimize model parameters to reach the best forecasting accuracy.

ANFIS is the major training routine for Sugeno-type fuzzy inference systems. In ANFIS proposed in [27], the advantages of Fuzzy logic (FL) and artificial neural networks (ANNs) were combined for adjusting the membership functions (MFs), the rule base and related parameters to fit the training dataset. ANFIS uses a hybrid learning algorithm to identify parameters of Sugeno-type fuzzy inference systems. It applies a combination of the least-squares method and the back-propagation gradient descent method for training FIS membership function parameters to emulate the given training data set. The parameters associated with the membership functions will change through the learning process. The training process stops whenever the maximum epoch number is reached or the training error goal is achieved. Readers are referred to [27, 28] for more details.

B. Tuning process

The tuning process involves starting from a previous knowledge base (rule base and data base) either derived by any learning method or provided by experts.

In our work, we assume the existing of a previous definition of an existing FRBS (provided by the learning process previously described). Further optimization of the existing fuzzy model deals with an adjustment of a suite of parameters (membership function parameters of input/output variables and rule consequents). This tuning process is based on Particle Swarm Optimization method (PSO) which aims to modify the shapes of the membership functions and adjust the parameters of the consequent parts of the fuzzy rules in order to improve the system performance.

The context of identification of fuzzy models using PSO algorithm can involves a number of important considerations. The first one is to define solution space (ranges of variables to be optimized), the fitness function and a set of constraints. Another important consideration is the solution encoding i.e. to represent a fuzzy model by a particle (a set of particles represent a population). Every particle in the search-space is basically representing a fuzzy model which consists of two parts: one represents membership functions of antecedents and consequents and second part represents rule-base. After every iteration, the performance of each fuzzy model is to be worked out to determine the movement of all the particles in the swarm.

The main idea of the approach is to generate a fuzzy inference system (FIS) from each particle. The FIS structure is an object that contains all the information about the fuzzy inference system i.e. membership function definitions, variables names, rule base etc.

A FIS has a structure that can be easily modified. This flexibility has been used for modifying the parameters of fuzzy models through PSO encoding mechanism.

C. Encoding mechanism of the fuzzy system

When we use PSO algorithm, a very important consideration is to completely represent a fuzzy system by a particle, and for this, all the needed information about the rule-base and membership functions is required to be specified through some encoding mechanism.

It is also suggested to modify the membership functions and rule-base simultaneously, since they are codependent in a fuzzy system.

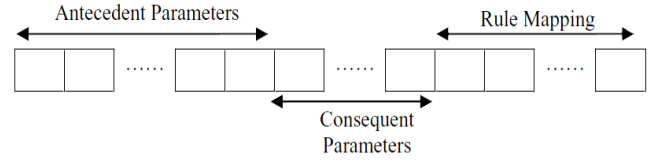


Fig. 3. Representation of a fuzzy model by a particle

One of the most important steps is to provide an efficient encoding method. In our work, we consider Gaussian membership functions defined by a central value c and the width σ . Each particle represents a fuzzy model. After definition of the initial structure for the fuzzy system, the model parameters, i.e., the widths of the Gaussian membership functions are previously determined by the learning method (subtractive clustering and ANFIS). Then, the second parameter c (center of every membership function) and the consequent value of every rule are encoded into a particle to be tuned via PSO.

Consequently, a particle representing a fuzzy model whose membership function parameters of input/output variables and rule consequents are optimized through PSO algorithm to follow the best fuzzy model.

D. Fitness Function

The fitness/objective function represents the quality of each solution and also provides a link between the optimization algorithm and the problem under consideration. The difference between computed output and actual output as given in the dataset gives the error. To find an optimal fuzzy inference system, the Root Mean Squared Error (RMSE) is employed as a fitness function:

$$Fit(i) = \sqrt{\frac{1}{P} \sum_{j=1}^P (y_t^j - y_{out}^j)^2} \quad (3)$$

where P is the total number of samples, y_t^j and y_{out}^j are the desired output and the predicted model output of j^{th} sample. $Fit(i)$ denotes the fitness value of i^{th} individual.

E. The hybrid evolving algorithm

The process for the identification of fuzzy model using our methodology is represented as pseudo-code as follows:

Step 1.

Define operating parameters for PSO algorithm;

Generate a random set of particles (initial population);

Step 2.

Derive initial FIS by learning methods (subtractive clustering and ANFIS) as described in section A. This step will decide the number of membership functions and fuzzy rules.

Step 3.

Set the generation sizes (G) and initialize $g = 0$.

Step 4.

Adjustment of a suite of parameters by the proposed PSO (membership function parameters of input/output variables and rule consequents) to derive the corresponding fuzzy modeling system.

Step 5.

Evaluate each particle for its fitness (RMSE) using (3), and then compare each particle's fitness value with best global particle value (Gbest) and the personal best value (Pbest). Select the new Gbest and Pbest.

Step 6.

For every particle, update its velocity and position value according to (1) and (2).

Step 7.

$$g = g + 1.$$

Step 8.

If $g = G$, then exit, otherwise go to step 4.

Step 9.

The best particle's value will be selected as the final parameter set to build the desired optimized fuzzy model.

V. EXPERIMENTAL RESULTS

To fully evaluate the performance of our methodology and the other algorithms, the well-known benchmark problems "Mackey-Glass chaotic", "Jenkins-Box" and "sunspot number" were employed. The best-suited sets of parameters employed in the simulation study are chosen after a series of tuning experiments. These parameters are listed in table 1.

A. Mackey-Glass time series prediction

The Mackey-Glass (MG) series, based on the Mackey-Glass differential equation [29], is often regarded as a benchmark used for testing the performance of neural network models and fuzzy systems. This series is a chaotic time series generated from the following time-delay ordinary differential equation:

$$\frac{d(x(t))}{dt} = \frac{ax(t-\tau)}{1+x^c(t-\tau)} - bx(t) \quad (4)$$

The setting of the experiment varies from one work to another. In this work, $a = 0.2$, $b = 0.1$, $c = 10$ and $\tau \geq 17$, were adopted. The task of this study is to predict the value of the time series at point $x(t+6)$, with using the inputs variables $x(t)$, $x(t-6)$, $x(t-12)$ and $x(t-18)$. 1000 sample points are used in our study. The first 500 data pairs of the series are used as training data, while the remaining 500 are used to validate the model identified.

We ran the simulation 10 times and averaged the results. After 50 generations ($g=50$), the final optimal fuzzy model was obtained with RMSE 5.0435e-005. The RMSE value for validation data set is 5.1162e-005. The actual time-series output and the optimized output of the best fuzzy system for training and testing data are shown in Figure 4 and Figure 5.

TABLE I. LIST OF INITIAL PARAMETERS

PSO	
Parameter	Initial value
Population size (NP)	20
c_1	0.2
c_2	0.6
subtractive clustering	
Parameter	Initial value
cluster radius (MG problem)	0.8
cluster radius (BJ problem)	0.6
cluster radius (sunspot number problem)	0.7

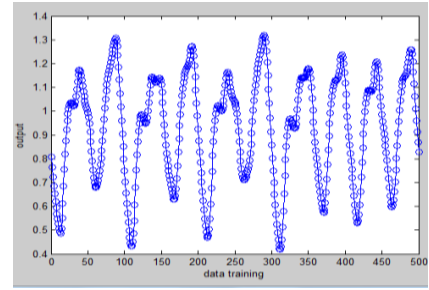


Fig. 4. The actual time series data and the calculated output of the fuzzy model for training data to forecast Mackey-Glass data

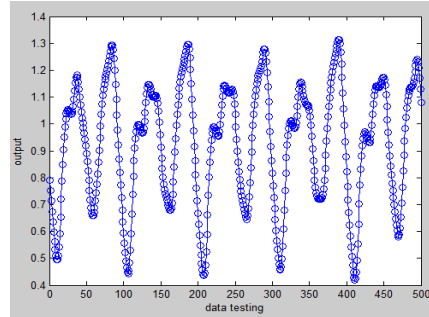


Fig. 5. The actual time series data and the calculated output of the fuzzy model for testing data to forecast Mackey-Glass data

The experimental results indicate that the proposed system performs well in terms of RMSE.

For a fair experimentation study, the proposed algorithm was compared with three other contributions. The first one described in [30] presented a design process of the TSK type fuzzy model using evolutionary algorithm and least square method. The second contribution [31] presented two other CI techniques, COPSO-SMN and ANFIS. The third one is described in [32] and it proposed a design of fuzzy inference system by means of FCM clustering algorithm and evolutionary optimization using real-coded genetic algorithm. Finally, contribution [33] outlines a fuzzy set based granular evolving modeling - FBeM - approach.

A comparison result of those different methods for forecasting Mackey-Glass data is shown in Table 2. It was observed that the proposed algorithm outperforms the other algorithms on RMSE performance basis and on the number of generations.

TABLE II. COMPARISON RESULTS FOR THE PREDICTION OF MACKEY-GLASS TIME-SERIES

Method	Generation	Training error (RMSE)	Testing error (RMSE)
[30]	300	0.0015	0.0014
COPSO-MSN [31]	500	0.3223	0.3243
ANFIS [31]	-	0.0064	0.0064
[32]	200	0.0005	0.0006
FBeM [33]	-	-	0.0122
The proposed algorithm	50	5.0435e-005	5.1162e-005

B. Box and Jenkins' Gas Furnace Problem

The Box-Jenkins dataset (BJ) [34] represents the CO2 concentration as output, $y(t)$, in terms of input gas flow rate into the furnace, $u(t)$, from a combustion process of a methane-air mixture. From a total set of 296 data pairs, first 200 data points were used for training and the remaining data samples are used for test. The aim is to predict $y(t)$ in terms of $y(t-1)$ and $u(t-4)$.

After 50 generations ($g = 50$), the optimal fuzzy model was obtained with the RMSE 0.0048. The RMSE value for validation data set is 0.0116. Figure 6 and 7 show the actual and the predicted time series for training and testing data. A comparison result of different methods for Jenkins-Box data prediction is shown in Table 3.

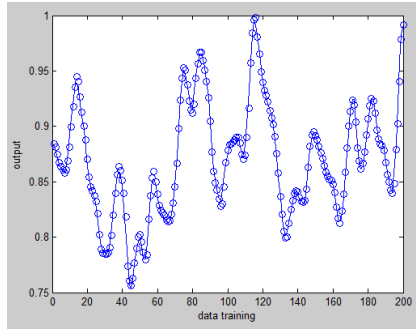


Fig. 6. The actual time series data and the calculated output of the fuzzy model for training data to forecast Box and Jenkins data

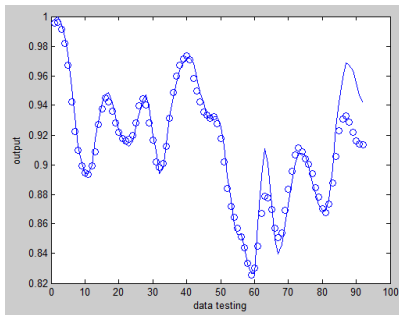


Fig. 7. The actual time series data and the calculated output of the fuzzy model for testing data to forecast Box and Jenkins data

TABLE III. COMPARISON RESULTS FOR THE PREDICTION OF BOX AND JENKINS TIME-SERIES

Method	Training error (RMSE)	Testing error (RMSE)
COPSO-MSN [31]	0.2151	0.3416
ANFIS [31]	0.0374	0.0640
FBeM [33]	-	0.0421
The proposed algorithm	0.0116	0.0048

C. Prediction of sunspot number time series

The sunspot number data set is the series of the sunspot annual average numbers. This series illustrates the yearly average relative number of sunspot observed [35].

Samples of data between 1700 and 1920 were used for training. Then, in order to validate the model, two other sets of data samples were used, the first one is from 1921 to 1955 and the second is from 1956 to 1979. The aim is to estimate the output at point $y(t)$, with using the inputs variables $y(t-4)$, $y(t-3)$, $y(t-2)$, $y(t-1)$.

After 50 generations ($g = 50$), an optimal fuzzy model was generated with RMSE 2.4154e-007. The RMSE value for the first data set validation is 2.7834e-007 and for the second data set validation is 4.7498e-007. Figures 8, 9 and 10 show the actual and the predicted time series for training and testing data (including the two test cases). A comparison between several techniques is illustrated in Table 4. As presented in this

table, the proposed methodology outperforms the other methods.

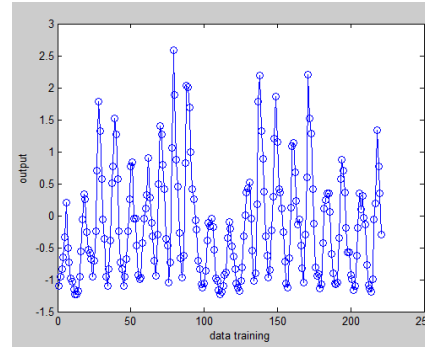


Fig. 8. The actual time series data and the calculated output of the fuzzy model for training data to forecast sunspot number time series

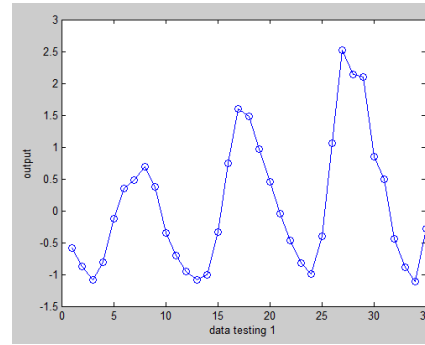


Fig. 9. The actual time series data and the calculated output of the fuzzy model for the first testing data to forecast sunspot number time series

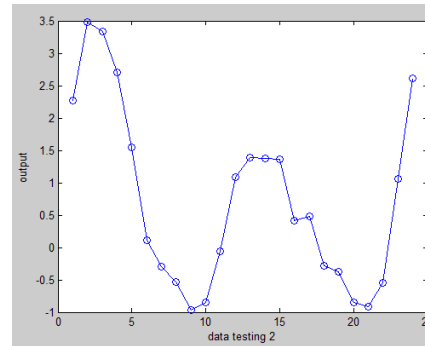


Fig. 10. The actual time series data and the calculated output of the fuzzy model for the second testing data to forecast sunspot number time series

TABLE IV. COMPARISON RESULTS FOR THE PREDICTION OF SUNSPOT NUMBER TIME SERIES

Method	RMSE Training	RMSE Testing 1	RMSE Testing 2
Transversal Net [36]	0.0987	0.0971	0.3724
Recurrent net [36]	0.1006	0.0972	0.4361
FWNN-R [37]	0.0796	0.1099	0.2549
FWNN-M [37]	0.0828	0.0973	0.1988
The proposed algorithm	2.4154e-007	2.7834e-007	4.7498e-007

VI. CONCLUSION

This paper proposes the application of some fast techniques like subtractive clustering and Adaptive Neuro Fuzzy Inference System (ANFIS) for creating an initial fuzzy model from an available data. Then, we apply Particle Swarm Optimization Algorithm (PSO) to adjust the parameters

(shapes) of membership functions and rule consequents. As a result, we obtain a Fuzzy Rule-Based System with a good accuracy. From the simulation results, we can affirm that this new hybrid technique proves its superiority over the other compared methods.

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