

# Multi-Agent Evolutionary Design of Beta Fuzzy Systems

Y. Jarraya, S. Bouaziz, Adel M. Alimi and A. Abraham

**Abstract**— This paper provides an overview on a new evolutionary approach based on an intelligent multi-agent architecture to design Beta fuzzy systems (BFSs). The Methodology consists of two processes, a learning process using a clustering technique for the automated design of an initial Beta fuzzy system, and a multi-agent tuning process based on Particle Swarm Optimization algorithm to deal with the optimization of membership functions parameters and rule base. In this approach, dynamic agents use communication and interaction concepts to generate high-performance fuzzy systems. Experiments on several data sets were performed to show the effectiveness of the proposed method in terms of accuracy and convergence speed.

## I. INTRODUCTION

At present, fuzzy system modeling is considered as one of the most important areas of application in fuzzy theory.

Fuzzy set, a concept proposed by Zadeh [1], has been widely investigated, due to its effectiveness in modeling the imprecision and uncertainty in human reasoning. Fuzzy logic is suitable for the representation of vague data and it provides an appropriate mechanism to describe the static and dynamic behavior of complex systems. In this sense, fuzzy systems are being used successfully in an increasing number of application areas such as classification [2]–[5], regression [6], [7], prediction [8], control problems [9]–[11], and general data mining problems [12], [13].

In general, fuzzy modeling includes identifying both the structure and the parameters of fuzzy systems. In this sense, intelligent optimization techniques such as artificial neural network [14],[15], clustering techniques [8], [13], and evolutionary computation [8], [16]–[20] have been successfully applied in this area in order to obtain the desired fuzzy model from the existing knowledge.

In this context, we introduce a novel methodology to design high-performance fuzzy systems especially Beta fuzzy systems (BFSSs) for which the fuzzy basis functions used are Beta functions [21]–[23]. The use of Beta function shows a better performance against the other kind of functions due to its large flexibility and its universal approximation capacity [21]–[23].

The new approach consists of two processes: a learning process and a multi-agent tuning process. In the first one, to approximate the desired output, the data are first clustered by Subtractive clustering algorithm. As a result, an initial structure of the model is obtained with a compact rule base. At this point, we use the optimization capability of Particle Swarm Optimization algorithm enhanced by a multi-agent architecture to tune the parameters of membership functions (shapes) and rules consequents. The basic idea here is to use the multi-agent architecture to improve the accuracy of the system and to accelerate the convergence behavior.

The proposed multi-agent system structure offers a decentralized model based on agents which are able to re-estimate their decisions if needed after negotiations. Information exchange takes place between the different agents to establish a unified decision. This kind of cooperative interaction improves the agent performances, and as a result, we obtain a Fuzzy Rule-Based System (FRBS) with high accuracy. On the other hand, different numbers of Function Evaluations (FEs) were noted in order to evaluate the speed of the proposed approach and to express its complexity.

The paper is planned as follows: Section II briefly introduces the Beta basis function. Section III provides a brief account of PSO algorithm. A description of the learning and tuning steps is detailed in section IV. Then, in section V, we propose a multi-agent architecture to improve the optimization capability of our system. For the purpose of illustration and validation of the approach, time series prediction datasets are used in section VI. Finally, conclusions are drawn in Section VII.

## II. THE BETA BASIS FUNCTION

The first idea of using the Beta basis function for the design of fuzzy systems was introduced by Alimi in 1997 [21]. This leads to the appearance of the Beta fuzzy systems. A Beta Fuzzy Logic System is an FLS where the Beta functions are chosen as membership functions of the input variables. The use of Beta function was chosen for many advantages [21]–[23], such as its large flexibility, its ability to generate more rich shapes (linearity, asymmetry, etc) and its universal approximation characteristics. The Beta basis function used in the BFS is defined by:

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$$\beta(x) = \beta(x, x_0, x_1, p, q) = \begin{cases} \left(\frac{x-x_0}{c-x_0}\right)^p \left(\frac{x_1-x}{x_1-c}\right)^q & \text{if } x \in ]x_0, x_1[ \\ 0 & \text{elsewhere} \end{cases} \quad (1)$$

Where  $p > 0, q > 0, x_0$  and  $x_1$  are real parameters, and:

$$c = \frac{px_1 + qx_0}{p+q} \text{ is the center of Beta function.}$$

Let  $\sigma = x_1 - x_0$ .  $\sigma$  is the width of the Beta function which can be seen as a scale factor for the distance  $x - c$ .

$$\text{So: } \begin{cases} x_0 = c - \frac{\sigma p}{p+q} \\ x_1 = c + \frac{\sigma q}{p+q} \end{cases} \quad (2)$$

Equations (1) and (2) imply:

$$\beta(x, c, \sigma, p, q) = \begin{cases} \left[1 + \frac{(p+q)(x-c)}{\sigma p}\right]^p \left[1 - \frac{(p+q)(c-x)}{\sigma q}\right]^q & \text{if } x \in \left]c - \frac{\sigma p}{p+q}, c + \frac{\sigma q}{p+q}\right[ \\ 0 & \text{elsewhere} \end{cases} \quad (3)$$

In the multi-dimensional case (dimension =  $d$ ), the Beta function is defined by:

$$\beta(x, c, \sigma, p, q) = \prod_{i=1}^d \beta_i(x_i, c_i, \sigma_i, p_i, q_i) \quad (4)$$

Alimi has shown that if we have a given continuous real function and for any arbitrary precision, there exists a Beta fuzzy basis function expansion that approximates it.

### III. PARTICLE SWARM OPTIMIZATION ALGORITHM

The Particle Swarm Optimization algorithm (PSO) developed by Kennedy and Eberhart [24] is a robust stochastic optimization algorithm based on swarm intelligence. It is inspired by the coordinated collective behavior of insects, birds and fish. PSO is based on swarm (group) behavior moving around in the search space in order to find the best particle (candidate solution).

The particles in the swarm co-operate. They exchange information about what they've discovered in the places they have visited. In each time step, each particle of the swarm is accelerated toward its  $p_{best}$  (personal best) and the  $g_{best}$  (global best) locations. The velocity of each particle  $i$  changes dynamically according to the following equation:

$$V_i(t+1) = w * V_i(t) + C_1 * R_1 * (p_{best} - X_i(t)) + C_2 * R_2 * (g_{best} - X_i(t)) \quad (5)$$

Having worked out a new velocity, the position of particle  $i$  is simply equal to its old position plus the new velocity:

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (6)$$

Where:

$V_i(t)$ : Velocity of particle  $i$  at iteration  $t$ .

$X_i(t)$ : Current position of particle  $i$  at iteration  $t$ .

$C_1$  and  $C_2$ : Constant weight factors.

$w$ : inertia weight.

$R_1$  and  $R_2$ : Random factors in the range  $[0,1]$ .

$g_{best}$ : Best position found by the neighbors of particle  $i$ .

$p_{best}$ : Best position achieved so long by particle  $i$ .

### IV. EVOLUTIONARY DESIGN OF BETA FUZZY SYSTEMS

Fuzzy modeling has the purpose of identifying the parameters of a fuzzy system in order to achieve a desired behavior. In this sense, we propose a new fuzzy modeling method based on Beta membership functions. In the first step, a fuzzy clustering technique is used to derive initial Beta membership functions and fuzzy rules from numerical data. Then, a multi-agent PSO tuning process is applied to refine a preliminary Knowledge Base working at two different levels: adjusting the membership functions parameters and the consequent values of fuzzy rules.

In this section, after detailing the used learning method, we present a centralized tuning process based on PSO (Figure 1). In the next section, we will see how this process can be extended and optimized by introducing a multi-agent approach.

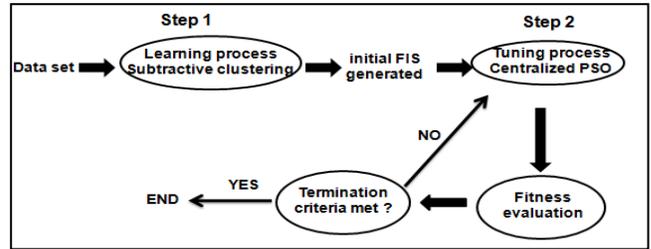


Fig. 1. Fuzzy modeling using centralized PSO as an optimization engine

#### A. Learning process based on subtractive clustering algorithm

One of the important tasks in fuzzy modeling is how to estimate fuzzy rules (structure identification). Generation of initial fuzzy rules can be done manually or automatically. In the first case, the expert's knowledge usually involves uncertainty and suffers from a loss of accuracy. In general, designers must spend a lot of time tuning fuzzy rules. In our work, instead of using a manual approach, a learning process is applied for the automated design of an initial structure of a fuzzy model from the available data. Here, the idea is to use an efficient clustering technique as the basis of a fast and robust method for fuzzy system identification.

Clustering plays a key role in searching for structures in Data. This technique is used to identify natural grouping of data from a large data set. It attempts to divide data into useful or meaningful subgroups named clusters. Therefore, the results must assure that samples which belong to the same cluster should be similar to one another and different from the samples of other clusters. In this context, we chose the subtractive clustering method by Chiu [25] to extract fuzzy rules from data. This algorithm is one-pass algorithm for estimating the number and location of cluster centers present

in a collection of data points. The center candidates correspond to the data samples themselves. This method works by finding the point with the highest number of neighbors as a center for a cluster based on the density of surrounding data points. Hence, each cluster center is translated into a fuzzy rule. As a result, it can be said that the use of such technique simplifies a lot the task of extracting fuzzy rules and reduces the computational effort. Moreover, the resultant fuzzy rules are more adapted to the input data than they are in a fuzzy system generated without clustering. This reduces the problem of large number of rules (combinatorial explosion of rules) when the input data have a high dimension.

On the other hand, it should be noted that the radius of influence of cluster center  $r_a$  constitutes an important parameter of the subtractive clustering algorithm. It strongly affects the number of clusters that will be generated. A small  $r_a$  can produce excessive number of clusters (resulting in many rules). Generally, specifying large  $r_a$  results in fewer clusters (resulting in fewer rules).

To summarize, in our research, the number of cluster centers is equal to the number of the fuzzy rules and the obtained cluster centers will constitute the Beta Membership functions centers. As a result, the initial structure of the fuzzy system will be built up and we obtain a compact rule base with a reduced number of rules. Since the proposed tuning method does not reduce the rule base size, this fact fits well to the design approach.

More details of Subtractive Clustering algorithm are presented in [25].

### B. Centralized tuning process based on Particle Swarm Optimization algorithm

In the tuning process, we start from a previous existing FRBS derived by the learning method previously described. In this level, PSO is applied for further optimization of the initial generated fuzzy inference system (FIS). The optimization process deals with a tuning of membership functions parameters and rule consequents in order to obtain the most accurate fuzzy system.

The context of fuzzy modeling involves an important consideration of how to encode the solution. In this work, the parameters and the rule base act as particles which look for the global best fitness. As shown in Figure 2, each particle of the swarm represents a fuzzy model (a population is represented by a set of particles). After every generation, we calculate the fitness function of each fuzzy model. Then, we update positions of particles through PSO algorithm until we follow the best fuzzy model.

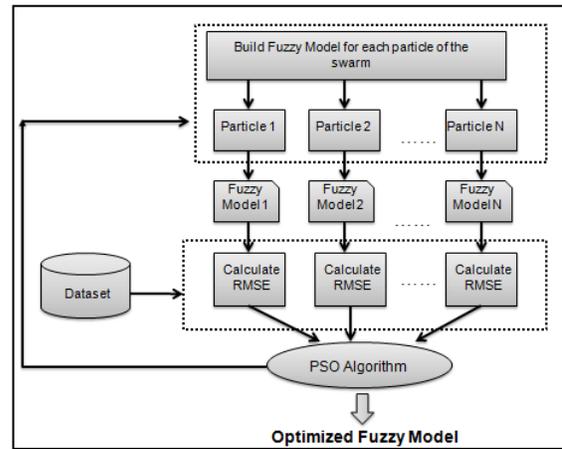


Fig. 2. Fuzzy Model Identification through PSO

The main idea here is to generate a FIS from each particle of the swarm. The FIS structure is an object containing all the information about the FIS, i.e. definitions of membership function, rule base, names of variables, etc. An FIS has a structure that can be easily modified. This flexibility has been exploited for optimization of the parameters and rule base using PSO encoding mechanism.

### C. Encoding mechanism of the fuzzy system

One of the most important steps is to provide an efficient encoding method. In most cases, triangular and Gaussian functions are usually used as fuzzy basis functions. However, we use the Beta function due to its large flexibility. This latter shows a better performance against the other type of functions.

Seen that the type of the membership functions used is the Beta function, four adjustable parameters must be considered in the learning and tuning process (the center  $c$ , the width  $\sigma$  and the form parameters  $p$  and  $q$ ). Subtractive clustering algorithm tunes only centers of membership functions  $c$ , and uses constant values of  $\sigma$ ,  $p$  and  $q$ . Consequently, the widths, the form parameters and the consequent values of rules are encoded into particles to be tuned via PSO as shown in Figure 3.

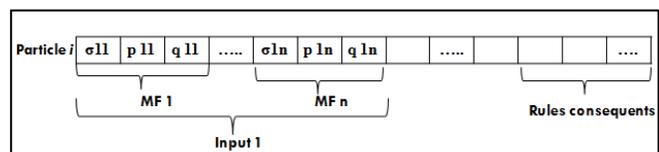


Fig. 3. Representation of a fuzzy model by a particle

To summarize, each particle of the swarm represents a fuzzy model whose membership function parameters and rule consequents are optimized through PSO algorithm to follow the best fuzzy model.

#### D. Fitness Function

The difference between the calculated output and the actual output (as given in the dataset) defines the error. In this work, the fitness of an individual is determined by calculating a biased Root Mean Squared Error (RMSE) of the fuzzy model:

$$Fit(i) = \sqrt{\frac{1}{p} \sum_{j=1}^p (y_t^j - y_{out}^j)^2} \quad (7)$$

where  $p$  is the total number of samples,  $y_t^j$  and  $y_{out}^j$  are respectively observed output and predicted output.  $Fit(i)$  defines the fitness function value of the  $i^{th}$  individual.

#### E. The centralized evolving algorithm

The process for the identification of fuzzy model using the centralized PSO is represented as pseudo-code by the following steps:

##### Step 1.

Generate an initial random population of candidate solutions.

##### Step 2.

Apply the learning process (subtractive clustering technique) to derive the initial structure of the fuzzy model as shown in section A. In this step, we will determine the number of Beta MFs and rules.

##### Step 3.

Tune the Beta MFs parameters ( $\sigma$ ,  $p$ ,  $q$  parameters) and the previously obtained rule base by applying the centralized tuning process through PSO algorithm (section B).

##### Step 4.

Evaluate the fitness value (RMSE) of each particle. Update individual and global best positions (Gbest and Pbest) by comparing the newly evaluated fitness against the previous individual and global best fitness.

##### Step 5.

For every particle:

- Calculate its velocity according to equation (5).
- Update its position according to equation (6)

##### Step 6.

Increment the generation number ( $g = g + 1$ ).

##### Step 7.

If  $g = G_{max}$  then exit, otherwise go to step 3.

##### Step 8.

The obtained best particle will be used to extract the final optimized fuzzy model.

## V. MULTI-AGENT ARCHITECTURE

A multi agent system is a set of entities that interact together to solve a problem or to reach a goal. In this work, the multi-agent approach is introduced in the level of the tuning process to improve the optimization performance of PSO algorithm. We attempt to develop an intelligent decision-making model based on multiple cooperative and negotiator agents to obtain the desired optimized fuzzy modeling system. Agents offer several potential advantages over the traditional centralized approach. Workers agents are not meant to work in isolation; they rather cooperate and are usually in a competitive interaction, and this allows a dynamic adaptation of their behavior. As a result, agents can re-estimate their decisions if needed during the negotiation process to avoid false convergence behavior. The main idea here is to affect to every agent its own PSO to be executed in order to generate its best solution (best particle) as shown in Figure 4. The initial population of each agent is randomly generated. In this context, we can introduce two types of communications:

- *Agent-Agent communication*

When executing its own PSO and after every generation, each agent worker sends its global best particle ( $g_{best}$ ) and the correspondent fitness value to the other agents. On the other hand, each worker will receive this  $g_{best}$  and will accept it for the next generation if and only if it yields a reduction in the value of the objective function. Otherwise, it will reject it.

This step of negotiation is expected to provide better convergence behavior of agents and also leads to a considerable reduction in convergence time.

- *Agent-Coordinator communication*

In the next step, when all agents have completed the execution of their PSO algorithms, the final best solution of each agent will be sent to the coordinator agent. This latter will form a new population by collecting all the best particles previously generated and then executes again a PSO algorithm. As a result, a Global best solution ( $G_{best}$ ) is obtained and then injected in the population of the least efficient agent (having the highest value of fitness function). This injection aims to repeat the same concept of parallelism until a satisfactory solution is found or a maximum global iteration number is reached.

To end, at each iteration, the new populations of agents will be formed as follows: the population of the least confident agent will contain the global best particle of the whole system  $G_{best}$  and the other particles will be liquidated at random. And for the other agents, their new populations will contain their own best particles and also random particles.

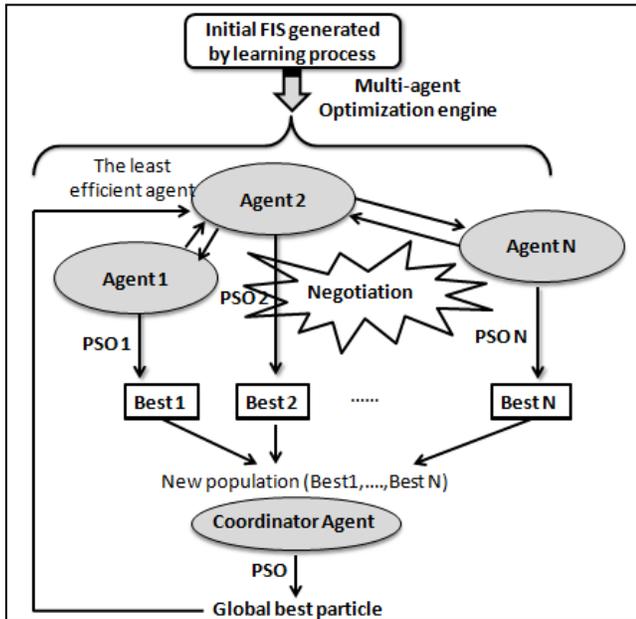


Fig. 4. General concept of fuzzy multi-agent tuning process

The aim of using a multi-agent architecture is to overcome the problems of premature convergence and slow of the classical centralized approach.

## VI. EXPERIMENTAL RESULTS

For the purpose of illustration and validation of the approach, the performance of the proposed method is evaluated for time series prediction datasets. A number of well-known benchmark problems such as “Mackey-Glass chaotic”, “Jenkins–Box” and “sunspot number” were employed. The best-suited sets of parameters used in the experimentation study are listed in table 1. We ran the simulation 10 times and then we averaged the results. The best solution in each case was marked in bold.

TABLE I. LIST OF INTIAL PARAMETERS

PSO	
Parameter	Initial value
Population size ( <i>NP</i> )	20
$c_1$	0.2
$c_2$	0.6
subtractive clustering	
Parameter	Initial value
cluster radius ( <i>MG</i> problem)	0.8
cluster radius ( <i>BJ</i> problem)	0.6
cluster radius ( <i>sunspot</i> number problem)	0.7

The tests results obtained are used to evaluate the effectiveness of the proposed method with respect to two measures of performance which are solution quality (through RMSE values) and convergence speed (through the number of Function Evaluations FEs marked). Accuracy and efficiency of the proposed methodology was demonstrated through comparisons with other available fuzzy/neural learning approaches developed in recent years.

### A. Mackey–Glass time series prediction

The Mackey-Glass series (MG) [26] is one of the well established benchmark problems which have been intensively studied in several previous works. This time series is based on the following Mackey–Glass differential equation:

$$\frac{d(x(t))}{dt} = \frac{ax(t-\tau)}{1+x^c(t-\tau)} - bx(t) \quad (8)$$

Parameters are selected as follows:  $a = 0.2$ ,  $b = 0.1$ ,  $c=10$  and  $\tau \geq 17$ . The aim is to predict the time series at point  $x(t + 6)$ . The inputs variables are respectively  $x(t)$ ,  $x(t - 6)$ ,  $x(t-12)$  and  $x(t-18)$ . In this study, 500 samples were selected for training and the next 500 samples were used for validating the identified model.

The obtained RMSE values for the training data and test data respectively are  $1.7902e-016$  and  $1.6883e-016$ . The number of Function Evaluations FEs used to find the optimum fuzzy model is equal to 56 which affirms the considerable reduction in convergence time. Figure 5 and 6 draw the actual and the forecast outputs for training and testing data.

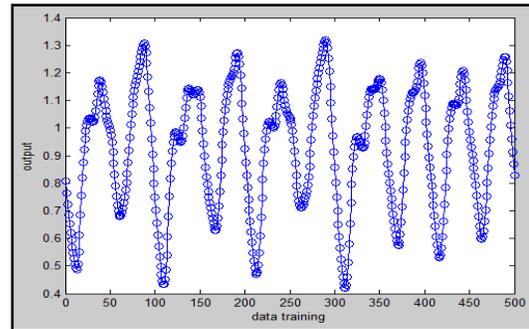


Fig. 5. The actual output (solid line) and the forecast output (circles) for training data in the case of Mackey-Glass

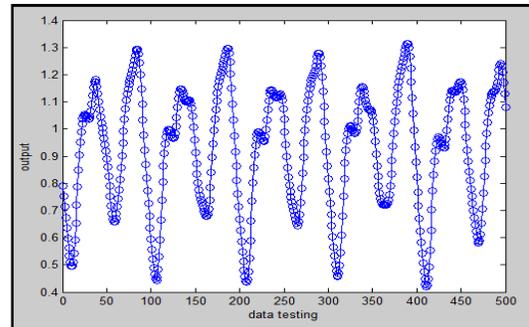


Fig. 6. The The actual output (solid line) and the forecast output (circles) for testing data in the case of Mackey-Glass

The experimental results indicate that the proposed system performs well in terms of accuracy and convergence speed. For a fair experimentation study, the results of the proposed method were compared with some of the previous studies.

The performance comparison to other works is given in Table 2. As observed, the prediction performance and accuracy by the proposed approach is much better than other compared approaches. But on the other hand, the number of FEs of those methods isn't mentioned.

TABLE II. COMPARISON RESULTS FOR THE PREDICTION OF MACKEY-GLASS TIME-SERIES

Method	Training error (RMSE)	Testing error (RMSE)
ADANN-GA [27]	0.055	-
ADANN-DE [27]	0.025	-
ADANN-EDA [27]	0.012	-
HCMSPSO [28]	0.0095	0.0208
HMDDE-BBFNN [29]	0.0094	0.0170
G-FNN [30]	0.0063	0.0056
HL-NFS [31]	0.0014	0.0013
NARMA [32]	0.000638	0.000627
LQFNM [33]	0.000543	-
FIS with non-uniform embedding [34]	0.0003497	-
FBONT_EIP& HBFOA [15]	5.3430e-10	1.8630e-09
<b>The proposed algorithm</b>	<b>1.7902e-016</b>	<b>1.6883e-016</b>

### B. Box and Jenkins' Gas Furnace Problem

The gas furnace data (BJ) [35] is also used as a benchmark problem. This time series problem is related to a combustion process of a methane-air mixture where CO<sub>2</sub> concentration in outlet gas is used as output,  $y(t)$ , and the input gas flow into the furnace is  $u(t)$ . From the 296 observations taken from a laboratory furnace [39], the first 200 were used for training and the remaining 96 were exploited for testing the proposed method. We aim to predict  $y(t)$  in terms of  $y(t-1)$  and  $u(t-4)$ .

After doing 98 number of Function Evaluations (FEs = 98), the optimal fuzzy model was generated with the RMSE 0.0049. For validation of data set, the RMSE value is 0.0125. Figures 7 and 8 show the actual and the predicted time series for training and testing data. Table 3 shows the comparison of test results of several models for Box-Jenkins data prediction problem. From this Table, it is clear that our method is having less training and testing errors in comparison to the other contributions.

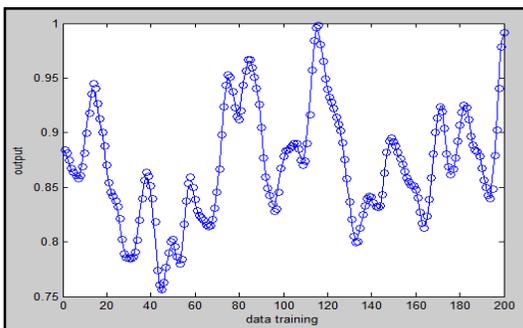


Fig. 7. The actual output (solid line) and the forecast output (circles) for training data in the case of Box and Jenkins

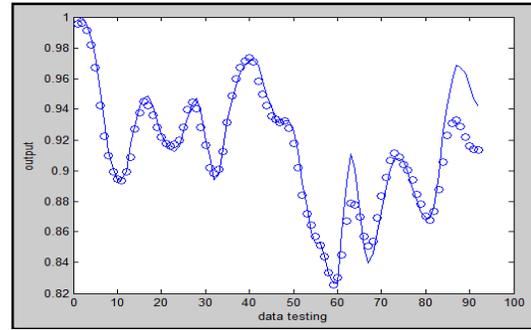


Fig. 8. The actual output (solid line) and the forecast output (circles) for testing data in the case of Box and Jenkins

TABLE III. COMPARISON RESULTS FOR THE PREDICTION OF BOX AND JENKINS TIME-SERIES

Method	Training error (RMSE)	Testing error (RMSE)
Subtractive clustering [36]	0.6134	-
KDE-based clustering [36]	0.6112	-
HMDDE-BBFNN [29]	0.2411	0.3745
HPSO [37]	0.2258	0.3876
COPSO-MSN [38]	0.2151	0.3416
ANFIS [38]	0.0374	0.0640
eTS [39]	-	0.04904
<b>The proposed algorithm</b>	<b>0.0049</b>	<b>0.0125</b>

### C. Prediction of sunspot number time series

The sunspot number time series corresponds to the annual average number of sunspots. This dataset represents the yearly average relative number of sunspot observed [40].

The  $y(t-4)$ ,  $y(t-3)$ ,  $y(t-2)$  and  $y(t-1)$  are used as inputs to the fuzzy system for the prediction of the output  $y(t)$ . Data points between 1700 and 1920 are used for system training. Then, two additional sets of data are employed for the test set. The first one is from 1921 to 1955 and the second is from 1956 to 1979.

After performing 78 Function Evaluations (FEs = 78), an optimal fuzzy model was obtained with RMSE 2.1316e-016. The RMSE value for the first data set validation is 5.6314e-005 and for the second data set validation is 3.1931e-005. From simulations and results, it's remarkable that our proposed method uses a reduced number of FEs to reach high accuracy solutions.

Figures 9, 10 and 11 illustrate the observed and the fitted time series for training and testing data (containing the two test cases). As presented in Table 4, a comparison is performed over a diverse collection of techniques. It is shown that the method proposed is competitive and robust in terms of accuracy and speed.

## VII. CONCLUSION

In this paper, an automated design of an initial Beta fuzzy system is first proposed by using subtractive clustering algorithm. Then, a tuning process is applied including a multi-agent Particle Swarm Optimization algorithm in order to tune the shapes of membership functions and rule consequents. This new hybridization aims to gather and make use of the clustering ability in extracting the knowledge base from data, the PSO ability to exchange social information, the high flexibility of Beta function and the robustness and convergence speed of the multi-agent architecture. The experimental results seem to be very encouraging as compared with other contributions and demonstrate the effectiveness of such an approach.

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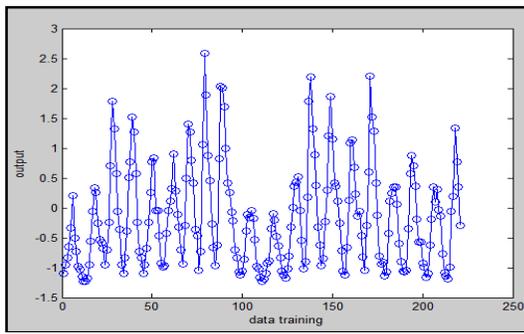


Fig. 9. The actual output (solid line) and the forecast output (circles) for training data in the case of sunspot number time series

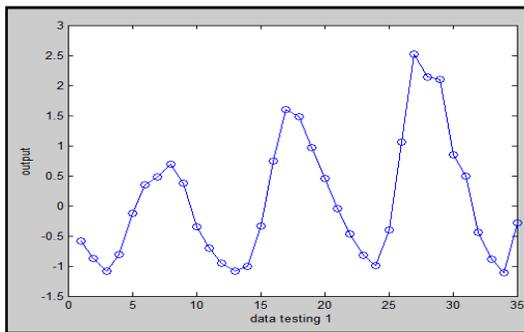


Fig. 10. The actual output (solid line) and the forecast output (circles) for the first testing data in the case of sunspot number time series

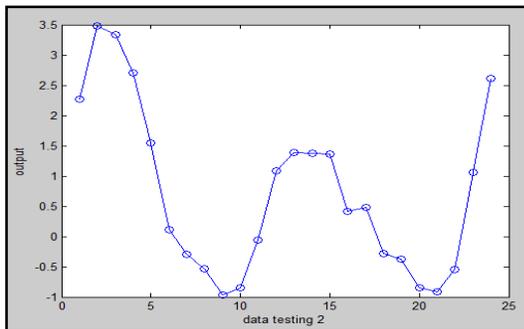


Fig. 11. The actual output (solid line) and the forecast output (circles) for the second testing data in the case of sunspot number time series

TABLE IV. COMPARISON RESULTS FOR THE PREDICTION OF SUNSPOT NUMBER TIME SERIES

Method	RMSE Training	RMSE Testing 1	RMSE Testing 2
FWNN-S [41]	0.0895	0.1093	0.1510
RFNN [42]	-	0.074	0.21
ABC-BBFNN [43]	0.0012	0.0018	0.0044
<b>The proposed algorithm</b>	<b>2.1316e-016</b>	<b>5.6314e-005</b>	<b>3.1931e-005</b>

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